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Test for Structural Change in Vector Error Correction Models[1]

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Abstract

In this paper we suggest a test for cointegration rank when change point is known and model has possibility that cointegration vector is changed. We consider possibility that cointegration rank is changed. In the test, we separate sample at given change point and estimate the model at twice for each sub sample. We use the estimation method that Johansen(1988,1991) proposed. This test statistics are based on the likelihood ratio. By the test we are able to test cointegration rank which is possibly changed. In addition, we are able to easily estimate the rank at changing cointegration vector and loading factor.

Keywords: structural break, cointegration, rank test.

1. Introduction

The cointegrated vector autoregressive (VAR) system is proposed by Johansen(1988,1991). The cointegrated VAR model is written by vector error correction model (VECM). The estimation of VECM is used for maximum likelihood estimation which is given cointegration rank and in which error independently and normally distributed. The cointegration rank is decided by likelihood ratio tests for cointegration rank.

There are some researches in cointegrated VAR model with structural break. Hansen and Johansen (1998) propose how to test for stability in VECM. The stability in VECM means that

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cointegrated vectors, conitegrated rank and adjusted coefficients are not changed. The stability is tested by recursive estimation for the eigenvalue and conitegration rank. If the model is stable, estimator of eigenvalue at full sample equal one at sub sample. In other research of stability, Quintos (1997) suggest a test. We are able to test stability by estimating recursively and taking max test statistic.

Seo (1999) suggest a test for structural change of cointegrated vector and loading factor. If structural change point is known, we are able to test for the change of cointegrated vector and loading factor by χ^2 test. If the change point is not known, we are able to test for structural change by estimating recursively. In case for recursive estimation for structural change, the cointegration rank is known.

A test which Hansen (2003) suggest have same condition. In this test, when the model has possibility that rank is change, we are able to test for structural change of cointegrated vector and loading factor. When we use the test, we must have the condition that change point and ranks is known.

There are some rank tests with structural change. In tests which Inoue (1999) and Saikkonen and Lutkepohl(2000) propose, the model has the condition in which deterministic trend is changed. Andrade, Bruneau, and Gregoir (2005) propose test which has structural change of cointegrated vector and loading factor. In these tests change point is known and unknown.

In this paper we suggest a test for cointegration rank when change point is known and model has possibility that cointegration rank is changed. We consider that all parameters in VECM is changed. In case of changing all parameters, we are able to separate the model by changing point. By using before and after changing sample, we estimate the model in twice. It is easy that we estimate for VECM with structural change by twice the estimation. In addition, since at each sub sample we estimate each rank, it is easy to consider rank changing. Test in this paper is likelihood ratio test. In condition that all parameters is changed we derive the rank test under way which Johansen (1988,1991) suggest.

In next section, we derive the cointegration model in structural change. Moreover, we derive the estimation which is maximum likelihood estimation. In section 3, we derive the test statistics on which rank changing is checked. Additionally, we consider hypothesis of test. In section 4, we derive asymptotic distribution and critical values of test statistics. In section 5, we give property of rank test by using Monte Carlo simulation.

2. Model and estimation

In this section, we derive the model for structural change in cointegration rank when change point is given. A cointegrated VAR model is written as the vector error correction model (VECM). Thus, we consider the VECM with the structural change.
In the first, we consider the cointegrated VECM without break.
The VECM follows that

$$\Delta X_t = \Pi\ X_{t-1} + \sum_{j=1}^{p} \Gamma_j \Delta X_{t-j} + \Phi D_t + \varepsilon_t \quad t = 1, \cdots, T,$$

which X_t is k × 1 and I(1), Π is k × k, ΠX_{t-1} is coitegrated relation and I(0), Γ_j is k × k, ε_t is k × 1 and normal as N(0, Ω) and independent and D_t is deterministic term. When deterministic term does not exist, is linear trend and quadratic trend, each D_t is 0, i and (i, t_q) which i=(1,⋯,1) and t_q=(1,⋯,T).

The cointegration rank is r. Π is written as $\Pi = \alpha\beta'$ and rank(Π) = rank(α) = rank(β) = r. In the second, we consider the structural break. In change point of t = T_b, the model happen the structural change. Thus, the model follows that

$$\Delta X_t = \Pi_1\ X_{t-1} + \sum_{j=1}^{p} \Gamma_{1j} \Delta X_{t-j} + \Phi_1\ D_t + \varepsilon_{1t} \quad t = 1, \cdots, T_b,$$

and

$$\Delta X_t = \Pi_2\ (X_{t-1} - X_{t_{b-1}}) + \sum_{j=1}^{p} \Gamma_{2j} \Delta X_{t-j} + \Phi_2\ D_t + \varepsilon_{1t} \quad t = T_b + 1, \cdots, T,$$

which rank(Π_i) = r_i, ε_{it} is normal as N(0, Ω_i) and independent. Π_i is written as $\Pi_i = \alpha_i\ \beta_i'$ and rank(Π_i) = rank(α_i) = rank(β_i) = r_i.

First period is t = 1, ⋯, T_b and second period is t = T_b + 1, ⋯, T.
In second period, we change X_{t-1} to $X_{t-1} - X_{t_{b-1}}$. If the model has deterministic trend, constant term increase only $-\Pi_2\ X_{t_{b-1}}$.

In this case, log likelihood which cointegration rank is given follows that

$$\log L\ (r_1, r_2, \theta) = \log L_1\ (r_1, \theta_1) + \log L_2\ (r_2, \theta_2)$$

which L(r_1, r_2, θ) and θ are each likelihood and parameter at full sample, L_1(r_1, θ_1) and θ_1 are each likelihood and parameter at first sub sample, and $L_2(r_2, \theta_2)$ and θ_2 each likelihood and parameter at second sub sample. Thus, maximum likelihood estimator at full sample is equal maximum likelihood estimator at each sub sample.

The model is written as

$$Z^1_{0t} = \Pi_1 Z^1_{1t} + \Psi_1 Z^1_{2t} + \varepsilon_{1t} \quad t = 1, \cdots, T_b,$$

and

$$Z^2_{0t} = \Pi_2 Z^2_{1t} + \Psi_2 Z^2_{2t} + \varepsilon_{2t} \quad t = T_{b+1}, \cdots, T,$$

which $Z^1_{0t} = \Delta X_t$, $Z^1_{1t} = X_{t-1}$, $Z^2_{1t} = X_{t-1} - X_{t_{b-1}}$, $Z^{i\prime}_{2t} = (\Delta X_{t-1}{}', \cdots, \Delta X_{t-p}{}', D_t{}')$ and $\Psi_i = (\Gamma_{i1}, \cdots, \Gamma_{ip}, \Phi_i)$.
R^i_{0t} and R^i_{1t} are residuals of regression each Z^i_{0t} and Z^i_{1t} on Z^i_{2t}.
We define S^1_{ij} and S^2_{ij} as

$$S^1_{ij} = \frac{1}{T_b} \sum_{t=1}^{T_b} R^1_{it} R^1_{jt} \quad \text{and} \quad S^2_{ij} = \frac{1}{T - T_b} \sum_{t = T_b + 1}^{T} R^2_{it} R^2_{jt},$$

$\hat{\lambda}^i_j$ is equal eigenvalue of matrix $(S^i_{11})^{-1} S^i_{10} (S^i_{00})^{-1} S^i_{01}$ which $\hat{\lambda}^i_1 > \hat{\lambda}^i_2 > \cdots > \hat{\lambda}^i_k$.
In addition, v^i_j is eigenvector associated $\hat{\lambda}^i_j$.
In case which cointegration rank is given r_1 and r_2, the estimator of cointegration vector follows that

$$\hat{\beta}_i = (v^i_1, \cdots, v^i_{r_i}),$$

The estimator of lording factor follows that

$$\alpha_i = S^i_{10} \hat{\beta}_i (\hat{\beta}_i{}' S^i_{00} \hat{\beta}_i)^{-1}$$

If the cointegration ranks are known, by using this method we estimate the VECM. But, usually cointegration ranks are not known. Thus, we have to estimate the ranks.

3. Test statistics and hypothesis

The test for rank of cointegration vector is based on likelihood ratio. In case of changing cointegration rank, we notice the selection the hypothesis. When rank under the null hypothesis is more than under alternative hypothesis, we are not able to test the rank.

Therefore, rank under the null hypothesis is less than under alternative hypothesis in each period. In first period rank of null and alternative hypothesis is each $r_1^0 \geq r_1$ and $r_1^a = r_1$. In second period rank of null and alternative hypothesis is each $r_2^0 \geq r_2$ and $r_2^a = r_2$. On hypothesis the test statistic follows that

$$-2\log LR(r_1^0, r_1^a, r_2^0, r_2^a, \tau) = -2\log LR_1(r_1^0, r_1^a) - 2\log LR_2(r_2^0, r_2^a)$$
$$= -T\tau \sum_{i=r_1^0}^{r_1^a} \log(1-\widehat{\lambda_{1t}}) - T(1-\tau)\sum_{i=r_2^0}^{r_2^a} \log(1-\widehat{\lambda_{2t}})$$

which $\tau = T_b/T$, LR is likelihood ratio at full sample, and LR_i is likelihood ratio at each sample. Since the likelihood ratio at full sample is equal summation of the likelihood ratio at each sub sample, we derive the test statistics to estimate at each sub sample. When rank is not changed, we are able to test trace test and max eigenvalue test in the same way without structural change. In this case, null hypothesis is that $r_1 = r_2 = r \geq r^0$, and alternative hypothesis is that $r_1 = r_2 = r^0 + 1$ or $r_1 = r_2 = k$. Usually, cointegration rank is not known. Thus, we start to test under the null hypothesis which is $r^0 = 0$. If null hypothesis is rejected, we again test under the null hypothesis which r^0 is added 1. We repeat by acceptance of null hypothesis.

When we consider possibility of changing rank, how to decide null and alternative hypothesis is a complicated question. We decide alternative hypothesis to take that $r_1 = r_1^0 + 1$, $r_2 = r_2^0 + 1$ or $r_1 = r_2 = k$ in the same way at no changing. When under null hypothesis which is $(r_1 \geq r_1^0, r_2 \geq r_2^0)$ we reject one, null hypothesis at next test is $(r_1 \geq r_1^0, r_2 \geq r_2^0 + 1)$ or $(r_1 \geq r_1^0, r_2 \geq r_2^0 + 1)$. If we select null hypothesis, in case of rejecting null hypothesis there is the same problem.

One solution is that under all combination r_1^0 and r_2^0 we test rank. In this case, we test at k^2 time. In other case, at each sub sample we estimate cointegration rank which $\widehat{r_1}$ and $\widehat{r_2}$ are estimated ranks. Under all combination $r_1^o \geq \widehat{r_1}$ and $r_2^o \geq \widehat{r_2}$, we test rank. In this way to test, we test at $(k-\widehat{r_1})(k-\widehat{r_2})$ times.

4. Asymptotic distribution

In this section, we derive the asymptotic distribution and the critical values. The test statistic limits to faction of Brownian motion. We are not able to derive probability process of Brownian motion without using simulation. Therefore, using Monte Carlo simulation, we derive critical values.

The test statistics is summation of one at each sub sample. Thus, asymptotic distribution of test statistics at full sample is summation of one at each sub sample. In each period, the model is standard VECM. Therefore asymptotic distribution of the test statistic is distributed as

$$-2\log LR(r_1^0, r_1^a, r_2^0, r_2^a, \tau) \rightarrow \sum_{i=r_1^0}^{r_1^a} \lambda_{1i} + \sum_{i=r_2^0}^{r_2^a 1} \lambda_{2i}$$

which λ_{1i} and λ_{2i} are eigenvalue of each

$$\int_0^1 dW_1\, B_1' \left(\int_0^1 B_1\, B_1'\, ds\right)^{-1} \int_0^1 B_1\, d\, W_1'$$

and

$$\int_0^1 dW_2\, B_2' \left(\int_0^1 B_2\, B_2'\, ds\right)^{-1} \int_0^1 B_2\, d\, W_2'$$

and $1 > \lambda_{j1} > \lambda_{j2} > \cdots > \lambda_{j(k-r_t)} > 0$.

In each case, B_i is different in deterministic terms. W_i is a standard Brownian motion which dimension is $k - r_i^0$ as follows that

$$W_i = (W_i^1, W_i^2, \cdots, W_i^{k-r_1^0})$$

In case 1, deterministic term does not exist at each period.
In this case $B_i = W_i$.

In case 2, the model has a constant and constant and loading factor is orthogonal.
In this case, $B_i = (W_i', 1)'$

In case 3, the model has a constant and constant and loading factor is not orthogonal.
In this case,

$$B_i = \left(W_i^1 - \overline{W}_t^1, W_i^2 - \overline{W}_t^1, \cdots, W_i^{k-r_1^0-1} - \overline{W}_t^{k-r_1^0-1}, s - \frac{1}{2}\right)$$

where

$$\overline{A} = \int_0^1 A\, ds.$$

In case 4, the model has a constant and trend and trend and loading factor is orthogonal.
In this case,

$$B_i = \left(W_i^1 - \overline{W}_t^1, W_i^2 - \overline{W}_t^1, \cdots, W_i^{k-r_1^0-1} - \overline{W}_t^{k-r_1^0}, s - \frac{1}{2}\right)$$

In case 5, the model has a constant and trend and trend and loading factor is not orthogonal. In this case,

$$B_i = (W_i^1 - a_t - b_i\, s, W_i^2 - - a_i - b_i\, s, \cdots, W_i^{k-r_1^0-1} - a_i - b_i\, s, s^2 - a - bs)$$

where a_i and b_i are constant and coeeficient in which W_i is regressed to s and a and b are constant and coefficients in which s^2 is regressed to s.

We calculate the critical values in only case[3] that contegration rank is not changed. In this case, the null hypothesis is $r_1^0 = r_2^0 = r$. The alternative hypothesis is $r_1^a = r_2^a = k$ in trace test and $r_1^a = r_2^a = r + 1$ in max eigenvalue test.We are not able to calculate critical values exactly. Therefore, using simulation of asymptotic distribution, we obtain critical values. The critical values were computed on a personal computer, using C code.

We simulate function of Brownian motion at 10000 times and approximate sample size which is 400 to large sample. We show the critical values for significant level 0.50, 0.20, 0.01, 0.05 and 0.01. Table 1 and table 2 show critical values at $\tau = 0.5$. Table 3 and table 4 show critical values at $\tau = 0.25$.

The critical values at $\tau = 0.5$ and $\tau = 0.25$ are about equal[4].

3 *We are able to easily calculate the critical value in the case the cointegration rank is changed. But there are too many tables. Thus the critical values in this case is omitted from this paper

4 These critical values are about equal one at $\tau = 0.4$ and $\tau = 0.3$.

Table1: Critical values of trace tests for $\tau = 0.5$

		$\alpha = 0.50$	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$
case1	k − r = 1	1.69	3.64	5.07	6.52	7.85	9.90
	k − r = 2	11.38	15.70	18.32	20.65	23.02	25.73
	k − r = 3	28.78	35.02	38.71	41.82	44.55	48.30
	k − r = 4	53.84	62.14	66.76	70.67	74.45	78.59
	k − r = 5	86.32	96.52	102.27	107.11	111.88	116.91
	k − r = 6	126.29	138.53	145.37	151.22	156.74	162.87
case2	k − r = 1	7.36	10.71	12.83	14.93	16.77	19.18
	k − r = 2	23.10	28.65	31.99	35.02	37.75	41.19
	k − r = 3	46.39	53.98	58.50	62.33	65.35	69.49
	k − r = 4	77.13	86.89	92.23	96.74	100.37	104.73
	k − r = 5	115.64	127.58	134.36	139.69	144.41	149.92
	k − r = 6	161.34	175.14	182.60	188.48	194.75	201.80
case3	k − r = 1	1.36	3.17	4.53	5.89	7.25	9.00
	k − r = 2	15.58	20.32	23.31	25.89	28.16	31.01
	k − r = 3	37.32	44.21	48.23	51.79	54.95	58.62
	k − r = 4	66.45	75.13	80.20	84.82	89.12	94.38
	k − r = 5	102.68	113.65	119.97	125.54	130.10	135.61
	k − r = 6	146.09	159.23	166.24	172.25	177.85	183.85
case4	k − r = 1	11.63	15.70	18.02	20.26	22.43	25.25
	k − r = 2	31.40	37.53	41.13	44.67	47.76	50.83
	k − r = 3	58.39	66.67	71.33	75.34	78.95	83.41
	k − r = 4	92.25	102.63	108.62	113.21	117.26	123.20
	k − r = 5	134.15	146.75	153.85	159.88	164.96	171.00
	k − r = 6	182.84	197.44	205.05	211.31	217.02	224.69
case5	k − r = 1	1.40	3.20	4.53	5.97	7.32	9.18
	k − r = 2	19.64	24.85	28.11	30.91	33.74	36.89
	k − r = 3	45.38	53.02	57.13	60.88	64.22	67.84
	k − r = 4	78.41	87.80	93.40	98.09	102.12	107.09
	k − r = 5	118.40	129.85	136.18	141.35	146.95	152.75
	k − r = 6	165.57	179.25	186.63	192.91	198.99	205.49

Table2: Critical values of max eigenvalue tests for $\tau = 0.5$

		$\alpha = 0.50$	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$
case1	k−r = 1	1.69	3.64	5.07	6.52	7.85	9.90
	k−r = 2	11.38	15.70	18.32	20.65	23.02	25.73
	k−r = 3	28.78	35.02	38.71	41.82	44.55	48.30
	k−r = 4	53.84	62.14	66.76	70.67	74.45	78.59
	k−r = 5	86.32	96.52	102.27	107.11	111.88	116.91
	k−r = 6	126.29	138.53	145.37	151.22	156.74	162.87
case2	k−r = 1	7.36	10.71	12.83	14.93	16.77	19.18
	k−r = 2	23.10	28.65	31.99	35.02	37.75	41.19
	k−r = 3	46.39	53.98	58.50	62.33	65.35	69.49
	k−r = 4	77.13	86.89	92.23	96.74	100.37	104.73
	k−r = 5	115.64	127.58	134.36	139.69	144.41	149.92
	k−r = 6	161.34	175.14	182.60	188.48	194.75	201.80
case3	k−r = 1	1.36	3.17	4.53	5.89	7.25	9.00
	k−r = 2	15.58	20.32	23.31	25.89	28.16	31.01
	k−r = 3	37.32	44.21	48.23	51.79	54.95	58.62
	k−r = 4	66.45	75.13	80.20	84.82	89.12	94.38
	k−r = 5	102.68	113.65	119.97	125.54	130.10	135.61
	k−r = 6	146.09	159.23	166.24	172.25	177.85	183.85
case4	k−r = 1	11.63	15.70	18.02	20.26	22.43	25.25
	k−r = 2	31.40	37.53	41.13	44.67	47.76	50.83
	k−r = 3	58.39	66.67	71.33	75.34	78.95	83.41
	k−r = 4	92.25	102.63	108.62	113.21	117.26	123.20
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	k−r = 4	78.41	87.80	93.40	98.09	102.12	107.09
	k−r = 5	118.40	129.85	136.18	141.35	146.95	152.75
	k−r = 6	165.57	179.25	186.63	192.91	198.99	205.49

Table3: Critical values of trace tests for $\tau = 0.25$

		$\alpha = 0.50$	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$
case1	k−r = 1	1.69	3.64	5.07	6.52	7.85	9.90
	k−r = 2	11.38	15.70	18.32	20.65	23.02	25.73
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	k−r = 2	31.40	37.53	41.13	44.67	47.76	50.83
	k−r = 3	58.39	66.67	71.33	75.34	78.95	83.41
	k−r = 4	92.25	102.63	108.62	113.21	117.26	123.20
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	k−r = 3	45.38	53.02	57.13	60.88	64.22	67.84
	k−r = 4	78.41	87.80	93.40	98.09	102.12	107.09
	k−r = 5	118.40	129.85	136.18	141.35	146.95	152.75
	k−r = 6	165.57	179.25	186.63	192.91	198.99	205.49

Table4: Critical values of max eigenvalue tests for $\tau = 0.25$

		$\alpha = 0.50$	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$
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	k − r = 2	11.38	15.70	18.32	20.65	23.02	25.73
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	k − r = 3	37.32	44.21	48.23	51.79	54.95	58.62
	k − r = 4	66.45	75.13	80.20	84.82	89.12	94.38
	k − r = 5	102.68	113.65	119.97	125.54	130.10	135.61
	k − r = 6	146.09	159.23	166.24	172.25	177.85	183.85
case4	k − r = 1	11.63	15.70	18.02	20.26	22.43	25.25
	k − r = 2	31.40	37.53	41.13	44.67	47.76	50.83
	k − r = 3	58.39	66.67	71.33	75.34	78.95	83.41
	k − r = 4	92.25	102.63	108.62	113.21	117.26	123.20
	k − r = 5	134.15	146.75	153.85	159.88	164.96	171.00
	k − r = 6	182.84	197.44	205.05	211.31	217.02	224.69
case5	k − r = 1	1.40	3.20	4.53	5.97	7.32	9.18
	k − r = 2	19.64	24.85	28.11	30.91	33.74	36.89
	k − r = 3	45.38	53.02	57.13	60.88	64.22	67.84
	k − r = 4	78.41	87.80	93.40	98.09	102.12	107.09
	k − r = 5	118.40	129.85	136.18	141.35	146.95	152.75
	k − r = 6	165.57	179.25	186.63	192.91	198.99	205.49

5. Monte Carlo simulation

In this section, we study cointegration test by Monte Carlo simulation. Specifically, we study whether the estimation of rank is equal to true rank. The cointegration test is preliminary test before VECM is estimated. Namely, we estimate the VECM using the result of rank test. Thus, we regard cointegration rank test as estimation method of cointegration rank test. We give the property of the cointegration rank estimation by the Monte Carlo simulation. This model in Monte Carlo simulation is

$$\Delta X_t = \Pi_1 X_{t-1} + \rho_1 + \varepsilon_{1t} \quad t = 1, \cdots, T_b,$$

and

$$\Delta X_t = \Pi_2 (X_{t-1} - X_{T_{b-1}}) + \rho_2 + \varepsilon_{2t} \quad t = T_b + 1, \cdots, T.$$

The cointegration rank is not changed[5] and the vector and the constant are changed. But it is difficult that we directly generate the X_t in this model. Thus we indirectly generate the X_t as follows that

$$z_t = z_{t-1} + e_t, \quad e_t \sim N(0, I_k), \quad t = 1, \cdots, T,$$

$$X_t = A_1 z_t + \rho_1 + u_t,\ u_t \sim N(0, I_k), \quad t = 1, \cdots, T_b,$$

and

$$X_t = A_2 z_t + \rho_2 + u_t,\ u_t \sim N(0, I_{k-r}), \quad t = T_b + 1, \cdots, T,$$

where z_t is $k - r \times 1$, A_i is $k \times k - r$. The number of the variables is $k = 6$, cointegration rank is $r = 0, 1, 2, 3, 4$ and 5, the number of sample is $T = 400$ and the break point is $T_b = 200$. In this Monte Carlo simulation, we estimate the cointegration rank. How to estimate the rank is that null hypothesis is that $r = r_0$, and alternative hypothesis is that $r = r_0 + 1$ or $r = k$, we start to test under the null hypothesis which is $r = 0$ and when null hypothesis is rejected, we again test under the null hypothesis which r_0 is added 1 and when null hypothesis is accept, estimation of rank is r_0. When we use this estimation method, without distortion proportion of false estimation to all estimation is significant level.

Table 5, 6 and 7 show proportion of true estimation of cointegration rank. In table 5, we estimate the cointegration rank by using section 3. Namely, in the summation test statistics after and before break point, we estimate the rank.

In table 6, we estimate the cointegration rank at each sub sample. Namely, we give after and

5 The cointegration rank is $r_1 = r_2 = r$.

before estimation of rank. Table 2 shows proportion that two estimations of rank is true. The test statistics at table 5 is summation of one at table 6.

In Table 7, we estimate the model which we do not consider the structural change. In other words, we estimate the misspecified model. The table of critical values we estimation rank test in table 5 is table 2. In table 6 and 7, we use usual table by Johansen without structural change.

Table 7 shows that the estimation of cointegration rank is mistaken. If model has cointegration, the model does not have cointegration.

Table 5 and 6 show that when sample is large estimation is effective. When sample is small, both are ineffective. Namely table 5 is more effective.

Table5: Monte Carlo simulation of rank estimation with structural break

		Trace test			Max eigenvalue test		
		$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
T = 100	r = 0	1.00	1.00	1.00	1.00	1.00	1.00
	r = 1	0.13	0.05	0.00	0.07	0.01	0.00
	r = 2	0.20	0.10	0.01	0.09	0.01	0.00
	r = 3	0.47	0.28	0.07	0.25	0.03	0.00
	r = 4	0.81	0.65	0.24	0.54	0.14	0.00
	r = 5	0.92	0.96	0.82	0.70	0.27	0.00
T = 150	r = 0	0.99	1.00	1.00	1.00	1.00	1.00
	r = 1	0.53	0.35	0.07	0.84	0.62	0.13
	r = 2	0.66	0.48	0.16	0.93	0.78	0.19
	r = 3	0.88	0.79	0.45	0.97	0.96	0.47
	r = 4	0.93	0.96	0.85	0.95	0.98	0.79
	r = 5	0.92	0.96	0.99	0.92	0.96	0.92
T = 200	r = 0	0.98	0.99	1.00	1.00	1.00	1.00
	r = 1	0.87	0.76	0.38	0.99	0.99	0.84
	r = 2	0.92	0.87	0.58	0.98	0.99	0.96
	r = 3	0.95	0.97	0.90	0.96	0.99	1.00
	r = 4	0.93	0.97	0.99	0.93	0.97	1.00
	r = 5	0.91	0.96	1.00	0.91	0.96	1.00
T = 300	r = 0	0.96	0.99	1.00	0.99	1.00	1.00
	r = 1	0.96	0.98	0.96	0.98	0.99	1.00
	r = 2	0.94	0.98	0.99	0.96	0.99	1.00
	r = 3	0.93	0.97	0.99	0.94	0.97	1.00
	r = 4	0.91	0.96	1.00	0.92	0.96	1.00
	r = 5	0.91	0.96	0.99	0.91	0.96	0.99
T = 400	r = 0	0.91	0.95	0.99	0.94	0.98	1.00
	r = 1	0.91	0.95	0.99	0.93	0.97	0.99
	r = 2	0.90	0.95	0.99	0.91	0.96	0.99
	r = 3	0.91	0.96	0.99	0.91	0.96	0.99
	r = 4	0.90	0.95	0.99	0.91	0.95	0.99
	r = 5	0.90	0.95	0.99	0.90	0.95	0.99

Table6: Monte Carlo simulation of rank separately estimation with structural break

		Trace test			Max eigenvalue test		
		$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
T = 100	r = 0	0.99	1.00	1.00	1.00	1.00	1.00
	r = 1	0.00	0.00	0.00	0.00	0.00	0.00
	r = 2	0.01	0.00	0.00	0.00	0.00	0.00
	r = 3	0.05	0.01	0.00	0.00	0.00	0.00
	r = 4	0.24	0.08	0.00	0.00	0.00	0.00
	r = 5	0.73	0.55	0.08	0.02	0.00	0.00
T = 150	r = 0	0.96	0.99	1.00	0.99	1.00	1.00
	r = 1	0.30	0.12	0.01	0.83	0.58	0.05
	r = 2	0.45	0.23	0.02	0.88	0.72	0.07
	r = 3	0.72	0.56	0.15	0.90	0.90	0.21
	r = 4	0.84	0.88	0.50	0.86	0.94	0.49
	r = 5	0.83	0.92	0.97	0.83	0.92	0.73
T = 200	r = 0	0.93	0.98	1.00	0.97	0.99	1.00
	r = 1	0.82	0.71	0.30	0.95	0.98	0.93
	r = 2	0.86	0.84	0.46	0.92	0.97	0.99
	r = 3	0.87	0.94	0.88	0.88	0.96	0.99
	r = 4	0.84	0.93	0.98	0.84	0.93	0.99
	r = 5	0.82	0.92	0.98	0.82	0.92	0.98
T = 300	r = 0	0.89	0.95	0.99	0.92	0.97	1.00
	r = 1	0.87	0.95	0.99	0.90	0.96	0.99
	r = 2	0.86	0.93	0.99	0.88	0.95	0.99
	r = 3	0.85	0.93	0.99	0.86	0.94	0.99
	r = 4	0.83	0.92	0.99	0.83	0.93	0.99
	r = 5	0.81	0.91	0.98	0.81	0.91	0.98
T = 400	r = 0	0.86	0.94	0.99	0.89	0.95	1.00
	r = 1	0.86	0.94	0.99	0.88	0.95	0.99
	r = 2	0.84	0.92	0.98	0.86	0.94	0.99
	r = 3	0.83	0.92	0.99	0.84	0.93	0.99
	r = 4	0.82	0.91	0.98	0.82	0.91	0.98
	r = 5	0.81	0.91	0.98	0.81	0.91	0.98

Table6: Monte Carlo simulation ofrank estimation not considering structural break

		Trace test			Max eigenvalue test		
		$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.01$
T = 100	r = 0	0.98	0.99	1.00	0.98	1.00	1.00
	r = 1	0.04	0.02	0.00	0.06	0.02	0.00
	r = 2	0.01	0.01	0.00	0.01	0.00	0.00
	r = 3	0.02	0.01	0.00	0.01	0.00	0.00
	r = 4	0.05	0.02	0.00	0.05	0.02	0.00
	r = 5	0.08	0.04	0.01	0.09	0.05	0.01
T = 150	r = 0	0.97	0.99	1.00	0.97	0.99	1.00
	r = 1	0.07	0.03	0.01	0.10	0.04	0.01
	r = 2	0.02	0.01	0.00	0.02	0.01	0.00
	r = 3	0.02	0.01	0.00	0.01	0.01	0.00
	r = 4	0.06	0.03	0.01	0.07	0.03	0.01
	r = 5	0.09	0.05	0.01	0.11	0.06	0.01
T = 200	r = 0	0.96	0.99	1.00	0.96	0.98	1.00
	r = 1	0.09	0.04	0.01	0.12	0.06	0.01
	r = 2	0.03	0.01	0.00	0.03	0.01	0.00
	r = 3	0.02	0.01	0.00	0.02	0.01	0.00
	r = 4	0.08	0.04	0.01	0.09	0.05	0.01
	r = 5	0.09	0.05	0.02	0.11	0.06	0.02
T = 300	r = 0	0.96	0.98	1.00	0.95	0.98	1.00
	r = 1	0.11	0.06	0.02	0.16	0.09	0.03
	r = 2	0.03	0.01	0.00	0.04	0.01	0.00
	r = 3	0.03	0.02	0.00	0.03	0.01	0.00
	r = 4	0.09	0.05	0.01	0.11	0.06	0.02
	r = 5	0.10	0.06	0.02	0.12	0.07	0.02
T = 400	r = 0	0.93	0.97	0.99	0.92	0.96	1.00
	r = 1	0.16	0.10	0.03	0.22	0.14	0.05
	r = 2	0.05	0.02	0.00	0.06	0.02	0.00
	r = 3	0.05	0.03	0.01	0.04	0.02	0.00
	r = 4	0.12	0.07	0.02	0.13	0.08	0.03
	r = 5	0.11	0.07	0.03	0.12	0.07	0.02

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