On a Selection Rule for utilizing the Component Selective Methods of Two-Dimensional Discrete Fourier Transform

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Index terms
two-dimensional DFT, subset of Fourier components,
Fourier feature extraction

Abstract

Generally, all sepectrum components are simultaniously calculated in two-dimensional fast Fourier transform (2-D FFT) even when only a previously specified subset of them is required. Five typical methods for calculating such a subset of components are introduced and the properties of them and the relations among them are discussed from a viewpoint of the computation costs.

One of those methods is effective in comparison with 2-D FFT method when $s \times t \leq N.(\log_2(M \times N) - \log_2 N)$. ($M, N$: the size of image data, $s \times t$: the number of specified components, $1 \leq s \leq M$, $1 \leq t \leq N$) Another method is always effective when $s \times t$ varies from 1×1 to $M \times N$. The latter is less effective than the former when $s \times t \leq (\log_2(M+N)) / (M \times N)$. The priority measure for the selection of the best method w. r. t. computation time is exactly made clear for $0 \leq (s \times t) / (M \times N) \leq 1$.

1 Introduction

Discrete Fourier analysis (DFA) is one of the most classical data processing techniques and the importance of DFA has been emphasized in many data processing fields. In order to make DFA practical, several kinds of improvements have been accomplished

w. r. t. more efficient FFT algorithms[1], easier design techniques of filters[2,3,4] and estimation methods of round-off error[5] or folding error[6].

Particularly in image data processing, other kinds of improvements are necessary as countermeasures to the extensive memory capacity due to 2-dimensional data. Onoe's method[7] which enables small scale computers to quickly process large scale image data and a new method for calculationg 2-dimensional Fourier series[8] are notable from this point of view.

This paper proposes a selection procedure for utilizing the component selective methods for calculating 2-dimensional discrete Fouries transform (2-D DFT). Five typical methods are introduced in this paper. In general, all spectrum components are simultaniously calculated in 2-dimensional fast Fourier transform (2-D FFT) even when only a subset of spectrum components are required. It is our idea to reduce the computation costs by calculating only the specified subset of components. The proposed procedure offers a technique to utilize the methods selectively w. r. t. their computation costs. Several utility restrictions against these methods and the degree of saved computation costs caused by them are also discussed in detail. It is also pointed out that they are applicable without any modifications to other orthogonal transforms and their inverse ones.

The component selective methods are introduced in section 2. Properties and relations of them are discussed in section 3, where the best method w. r. t. computation costs is exactly made clear. Other additional considerations are given in sections 4 and 5, respectively.

2 Preparations and component selective methods

2. 1 Preparations

Let $\mathbf{f} = \{f_{ij}\}$ ($i=0, 1, \ldots\ldots, M-1$, $j=0, 1, \ldots\ldots, N-1$; f_{ij}: real) be an image data and $\mathbf{F} = \{F_{kl}\}$ ($k=0, 1, \ldots\ldots, M-1$, $l=0, 1, \ldots\ldots, N-1$) be 2-D DFT of the image data $\mathbf{f}$. Each f_{ij} and F_{kl} are simply called an (picture) *element* and a (spectrum) *component*, respectively. 2-D DFT **F** is defined by eq. (1) and is easily modified as shown in eq. (2), where the 2-dimensional transform is divided into two steps of 1-dimensional discrete Fourier transform (1-D DFT). This process of transformation is denoted as **f** ⟶ **F'** ⟶ **F** hereafter.

$$F_{kl}=\sum_{j=0}^{N-1}\sum_{i=0}^{M-1} f_{ij}\ \exp\left\{-2\pi J\left(\frac{ki}{M}+\frac{lj}{N}\right)\right\},\ J=\sqrt{-1} \qquad (1)$$

$$k=0,\ 1,\ \ldots\ldots,\ M-1, \qquad l=0,\ 1,\ \ldots\ldots,\ N-1$$

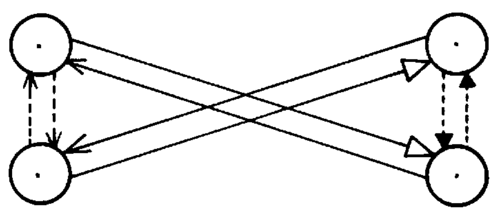
$$F_{kl}=\sum_{j=0}^{N-1}\underbrace{\left[\sum_{i=0}^{M-1} f_{ij}\ \exp\left\{-2\pi J\frac{ki}{M}\right\}\right]}_{=F'_{kj}}\exp\left\{-2\pi J\frac{lj}{N}\right\}$$

$$=\sum_{j=0}^{N-1}\left[F'_{kj}\right]\exp\left\{-2\pi J\frac{lj}{N}\right\} \qquad (2)$$

As 1-D DFT can be replaced by 1-dimensinal fast Fourier transform (1-D FFT), there exist formally five procedures for realizing 2-D DFT. One of them is such a procedure that each one of M×N components is directly calculated by means of difinition (1). (*direct-mode*) Other four procedures are DFT-mode, FFT-mode and *mixed-mode* I, II defined as eq. (3). (Also refer to Table 1.)

(3). (also refer to Table 1)

Table 1 Four confignrations for 2-D FFT with 1-D FFT

	1-D DFT	1-D FFT
VERTICAL TRANSFORM		
HORIZONTAL TRANSFORM		

⊙ :1dimensinal Fourier calculation

-----▷ :DFT-mode ---➧ FFT-mode

——> :mixed-mode I ——▷ :mixed-mode II

$$\left.\begin{array}{llll}
\text{direct-mode} & & \mathbf{f} \xrightarrow{\text{①}} \mathbf{F}\ ; & \text{①}:\text{2-D DFT} \\
\text{DFT-mode} & : & \mathbf{f} \xrightarrow{\text{①}} \mathbf{F'} \xrightarrow{\text{②}} \mathbf{F}; & \text{①, ②}:\text{1-D DFT} \\
\text{FFT-mode} & : & \mathbf{f} \xrightarrow{\text{①}} \mathbf{F'} \xrightarrow{\text{②}} \mathbf{F}\ ; & \text{①, ②}:\text{1-D FFT} \\
\text{mixed-mode I} & : & \mathbf{f} \xrightarrow{\text{①}} \mathbf{F'} \xrightarrow{\text{②}} \mathbf{F}\ ; & \text{①} : \text{1-D FFT} \\
 & & & \text{①} : \text{1-D DEF} \\
\text{mixed-mode II} & : & \mathbf{f} \xrightarrow{\text{①}} \mathbf{F'} \xrightarrow{\text{②}} \mathbf{F}\ ; & \text{②} : \text{1-D DFT} \\
 & & & \text{②} : \text{1-D FFT}
\end{array}\right\} \quad (3)$$

2. 2 Component selective methods of 2-D DFT

2-D DFT is a linear transformation from $M \times N$ elements to $M \times N$ components. Let the number of components be restricted to $s \times t$ ($1 \leq s \leq M$, $1 \leq t \leq N$) as shown in Fig. 1.+

Three principles are taken for the realization of the component selective methods (*CS-methods*) for calculating 2-D DFT.

① 2- DFT is divided into serial two steps of 1-dimensional transforms.

② 1-D DFT and 1-D FFT algorithms are combined as the serial two steps of 1-dimensional transforms in order to quickly calculate only the specified subset of components.

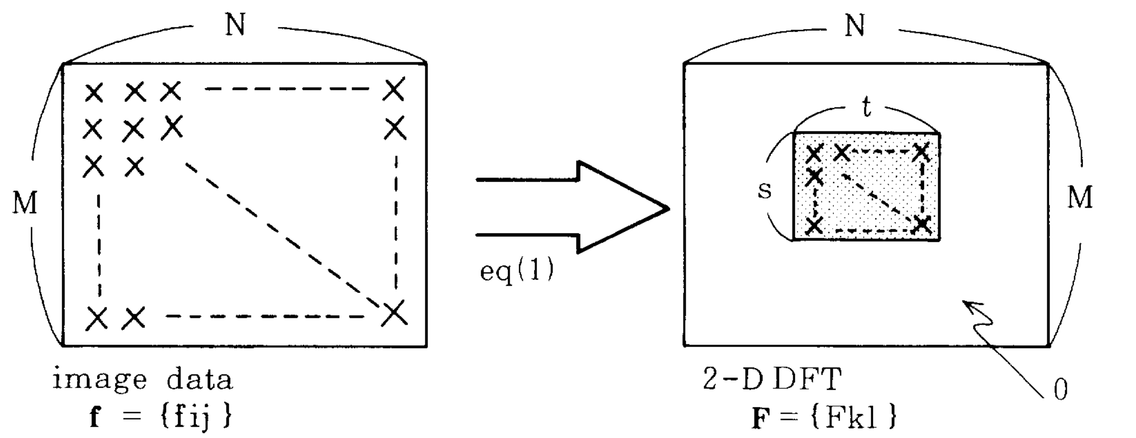


Fig. 1 A method for calculating only a subset of Fourier spectrum components

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+ It is supposed in this paper that the specified subset of components is a rectangular subregion of Fourier domain. This assumption scarcely provides the loss of generalities w. r. t. the evaluation of computation costs of new methods proposed here.

③ Both 1-D DFT and FFT for a line or row of image data can be omitted when nothing of the specified components is involved in the corresponding line or row in order to reduce the computation costs.

Application of these principles to five modes in eq. (3) produces next five *CS-methods* for calculating 2-D DFT. (*direct-selective, DFT-selective, FFT-method, mixed-method* I and II)

Procedures and computation costs of them are described by using next notations.

① $\mathcal{N}^{(k)}$ (M, N; s, t) : the number of required multiplications when only the $s \times t$ components are calculated from image data with $M \times N$ elements

② k : the index of the methods, k=1 (*direct-method*), k=2 (*DFT-method*), k=3 (*FFT-method*), k=4, 5 (*mixed-method* I, II)

③ log M : the logarithm of M to the base 2

(*i*) *mixed-method* I As N. MlogM multiplications for N 1-D FFT's ($\mathbf{f} \longrightarrow \mathbf{F}'$) and s. (t. N) multiplications for s 1-D DFT's ($\mathbf{F}' \longrightarrow \mathbf{F}$) are needed as shown in Fig. 2, the total number of multiplications is evaluated as eq. (4).+

$$\mathcal{N}^{(4)}(M, N; s, t) = (M \times N) \log M + (s \times t) . N \quad (4)$$

(*ii*) *other four methods* Similarly, $\mathcal{N}^{(k)}$ (M, N; s, t), k=1, 2, 3 and 5 are evaluated as shown in eqs. (5), (6), (7) and (8).

$$\mathcal{N}^{(1)}(M, N; s, t) = (s \times t) . (M \times N) \quad (5)$$

$$\mathcal{N}^{(2)}(M, N; s, t) = s. (M \times N) + (s \times t) . N \quad (6)$$

$$\mathcal{N}^{(3)}(M, N; s, t) = (M \times N) \log M + s. (N \log N) \quad (7)$$

$$\mathcal{N}^{(5)}(M, N; s, t) = s. (M \times N) + s. N \log N \quad (8)$$

+ If horizontal transformation precedes vertical one, we may exchange M with N and s with t in eq. (4), and vice-versa. In this case, $\mathcal{N}^{(4)}$ becomes $(M \times N) \log N + (s \times t) . M$. This exchange rule is valid also for $\mathcal{N}^{(1)}$, $\mathcal{N}^{(2)}$, $\mathcal{N}^{(3)}$ and $\mathcal{N}^{(5)}$.[9]

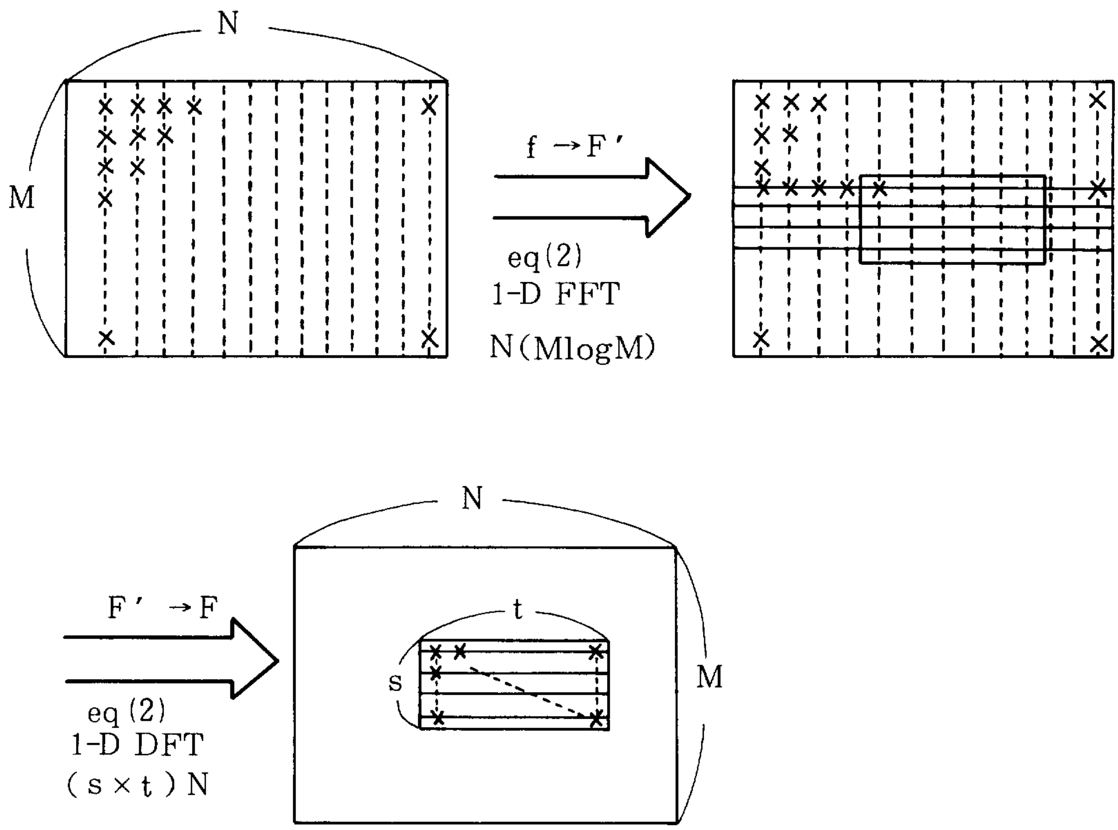


Fig. 2　Procedure of mixed-method I and the estimated number of multiplications

If let s be equal to M and t equal to N, *FFT-method* just agrees with 2-D FFT, where $\boldsymbol{N}^{(3)}$ (M, N; M, N)=(M×N) log (M×N). As the 2-D FFT method is a method which is commonly used on the real occasion, our subject is to clarify the properties of CS-methods and the relations among them by comparing $\boldsymbol{N}^{(1)}$, $\boldsymbol{N}^{(2)}$, $\boldsymbol{N}^{(3)}$, $\boldsymbol{N}^{(4)}$ and $\boldsymbol{N}^{(5)}$ with $\boldsymbol{N}^{(3)}$ $(M, N; M, N)$. These considerations are given in section 3.

2. 3　Generalizazion of CS-methods

CS-methods are generalized by putting such a constraint as shown in Fig. 3 and eq. (9) also to image data $\mathbf{f}=\{f_{ij}\}$. In this case, all elements are zero except for a subregion m×n of the whole region M×N. $(1 \leq m \leq M,\ 1 \leq n \leq N)$

$$\mathbf{f}=\{f_{ij}\},\quad f_{ij}\begin{cases}\neq 0: (i,\ j) \in m\times n \text{ subregion} \\ =0: (i,\ j) \in \text{elsewhere}\end{cases} \tag{9}$$

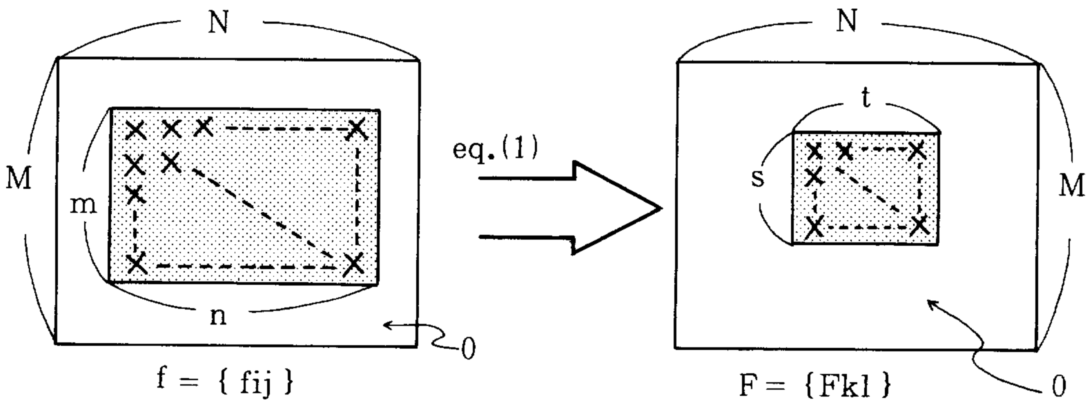


Fig. 3 Generalized method for calculating only a subset of Fourier components

New CS-methods generalized in this situation are called *generalized CS-methods* and are realized by omitting 1-D DFT or 1-D FFT calulations in lines or rows where all elemens take value 0.

The reasons why this constraint is practical in real systems are as follows:

① Two dimensional filters for image processing are usually designed as FIR filters[3] where the majority of elements take value 0. These filters are important for image processing.

② A frame which consists of zero-value elements is usually attached to the peripheral part of image data when $M < 2^a$ and / or $N < 2^b$ (a, b = 1, 2, 3, ...).

③ 2-D inverse FFT is frequently applied to bandlimited inputs where the majority of components take value zero. (Also refer to section 4. 1.)

(i) Generalized mixed-method I As n×MlogM multiplications for n 1-D FFT's (**f** ⟶ **F'**) and s × (t · N) multiplications for s 1-D DFT's (**F'** ⟶ **F**) are needed, as shown in Fig. 4, the total number of multiplications is evaluated by eq. (10).

$$\boldsymbol{N}^{(4)}(m, n; s, t) = n \cdot M\log M + (s \times t) \cdot N \qquad (10)$$

(ii) Other generalized CS-methods Similarly, $\boldsymbol{N}^{(k)}$ (m, n; s, t), k = 1, 2, 3 and 5 are evaluated and these results are summarized in Table 2, together with $\boldsymbol{N}^{(4)}$.

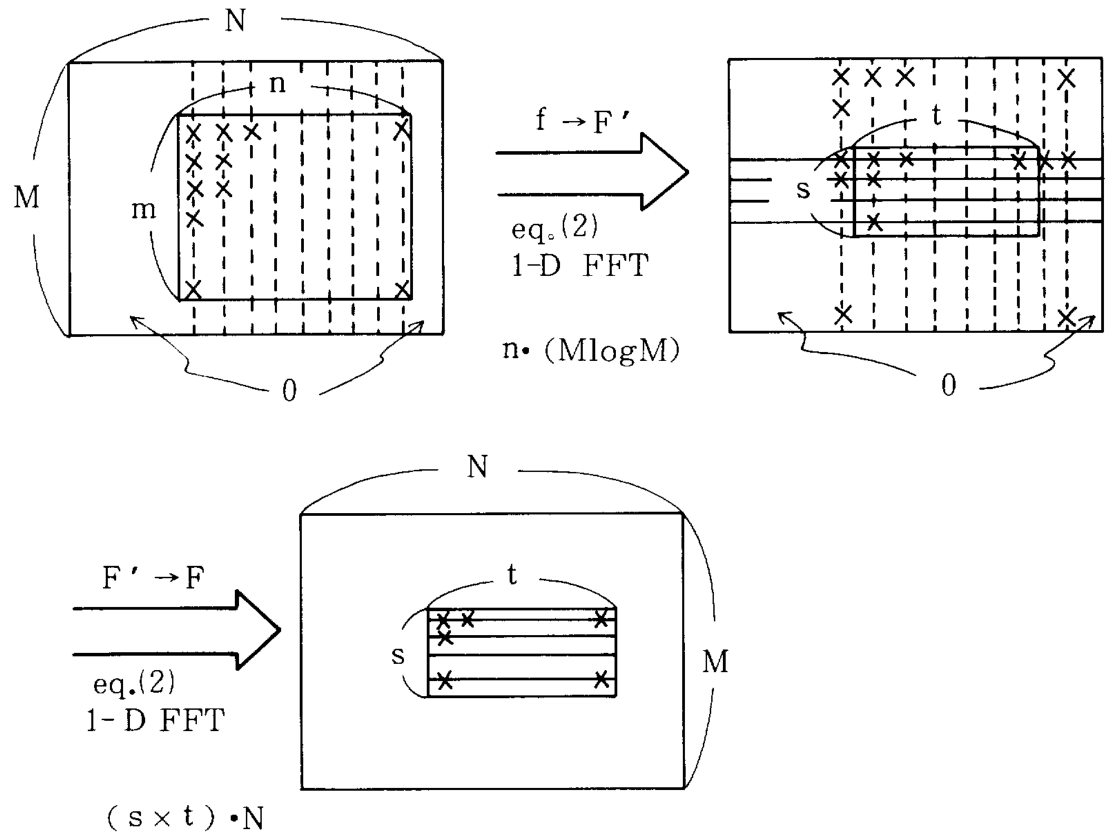


Fig. 4　Procedure of generalized mixed - method I and the estimated number of multiplications

Table 2　Theoretical computing costs

generalized CS - method	k	$N^{(k)}$ (m, n; s, t)	$E^{(k)}$ (Q) = $N^{(k)}/N^{(3)}$ (M, N; M, N)
direct - method	1	$(s\times t)\cdot(M\times N)$	$Q\cdot(M\times N)/\log(M\times N)$
DFT - method	2	$(n\times s)\cdot M+(s\times t)\cdot N$	$R\cdot(n\times M)/N\log(M\times N)$
FFT - method	3	$n\cdot M\log M+s\cdot N\log N$	$(n\cdot\log M)/(N\log(M\times N))+R\cdot\log N/\log(M\times N)$
mixed - method I	4	$n\cdot M\log M+(s\times t)\cdot N$	$n\cdot\log M/(N\log(M\times N))+Q\cdot N/\log(M\times N)$
mixed - method II	5	$(n\times s)\cdot M+s\cdot N\log N$	$R\cdot((m\times n)+N\log N)/(N\log(M\times N))$

- $Q=(s\times t)/(M\times N)$,　$R=s/M$,　$1\leq s,\ m\leq M$,　$1\leq t,\ n\leq N$
- This table is valid also for *CS - methods* when $m\to M$, $n\to N$.

These results concerning *generalized CS-methods* in Table 2 are valid also for *CS-methods* on the condition that m and n are exchanged with M and N, respectively.

3 Properties and relations of CS-methods

3. 1 Preparatory considerations

In the case of direct-method and mixed-method I, for example, eqs. (11) and (12) are derived based on the relations among $N^{(k)}$ (M, N; s, t), $N^{(3)}$ (M, N; M, N), k = 1, 4 as the upper bounds of s×t.

$$s \times t \leq \log (M \times N) \qquad : \text{direct-method} \qquad (11)$$

$$s \times t \leq M \cdot (\log (M \times N) - \log M) \qquad : \text{mixed-method I} \qquad (12)$$

Table 3 shows numerical examples of eqs. (11) and (12). It is known that mixed-method I for M = N = 128 can compute more than 64 times of components than direct-method. This means that components in lower band from 0-th to 15-th harmonics can be calculated by mixed-method I while only those from 0-th to 4-th harmonics are calculated by direct-method. These preparatory considerations lead us to the certainty that these CS-methods or generalized ones would be actually utilized in several applications.

In order to proceed these considerations systematically, next notations are used in the following discussions. Q in eq. (13) is the component selection ratio (*cs-ratio*) and $E^{(k)}$ (Q) in eq. (14) is the efficiency measure (*E-measure*) for k-th method.

Table 3 Numerical example of the upper bounds for s × t

M × N		8 × 8	16 × 16	32 × 32	64 × 64	128 × 128
direct-method		6	8	10	12	14
		9.4%	3.1%	0.98%	0.29%	0.08%
mixed-method I		24	64	160	384	896
		38%	25%	16%	9%	6%

upper column : the number of components s × t
lower column : cs-ratio (s × t) / (M × N) %

$$Q=(s\times t)\ /\ (M\times N) \tag{13}$$

$$E^{(k)}(Q)=\mathcal{N}^{(k)}(m,\ n;\ s,\ t)\ /\ \mathcal{N}^{(3)}(M,\ N;\ M,\ N) \tag{14}$$

, where $k=1, 2, 3, 4, 5$ and $\mathcal{N}^{(k)}$ (m, n; s, t) are modified by using Q as shown in Table 2.

3. 2 Properties of CS－methods

(*i*) *Permissible cs－ratio* When the efficiency measure $E^{(k)}(Q)$ is less than 1 as shown in eq. (15), CS－methods or generalized ones can be executed faster than 2－D FFT. Let Q_k be the maximum of Q which satisfies eq. (15) for each k and be called *permissible cs－ratio.*

$$Q_k=\max\ (Q)\ \text{s. t.}\ \ E^{(k)}(Q)\leq 1,\ \ k=1,\ 2,\ 3,\ 4,\ 5 \tag{15}$$

Q_1 for direct－method, Q_2 for DFT－method, Q_3 for FFT－method and Q_4, Q_5 for mixed－method Ⅰ, Ⅱ are easily derived from eq. (15) as a explicit functions of M, N, m and n, and they are summarized in table 4, where numerical examples are also added.

Table 4　Estimated permissible CS－ratio

k	permissible cs－ratio for k－th method (evaluated results)	M, N = 128	
		n = 128	n = 64
Q_1	$\log(M\times N)\ /\ (M\times N)$	0.08%	0.08%
Q_2	$(-n\cdot M+\sqrt{n^2\cdot M^2+4N^2\ M\log(M\times N)})^2\ /\ (4M\cdot N^3)$	0.99%	8.23%
Q_3	$Q_3\geq 1$	——	——
Q_4	$(N\cdot\log(M\times N)-n\cdot\log M)\ /\ N^2$	5.47%	8.21%
Q_5	$(M\times N)\ (\log(M\times N))^2\ /\ (n\cdot M+N\cdot\log N)^2$	1.08%	3.89%

According to these results, it is known that the relation among the permissible－ratio's is represented in eq. (16) for any pair of M and N.

$$Q_3\ (\text{FFT－method})\geq 1\geq Q_4\ (\text{mixed－method Ⅰ})$$

$$\geq Q_5\ (\text{mixed－method Ⅱ})\ \geq Q_2\ (\text{DFT－method}) \tag{16}$$

$$\gg Q_1\ (\text{direct－method})^{+}$$

＋ Because direct－method (or generalized one) scarcely has the practical efficiency as a component selective method for 2－D DFT, let us put it out of considerations hereafter.

It is also convincing that at least one CS-method or generalized one exists for any specified cs-ratio Q within $0 \leq Q \leq 1$.

(*ii*) *characteristics of* $E^{(k)}$ (Q) *measures* In Fig. 5 and Fig. 6,

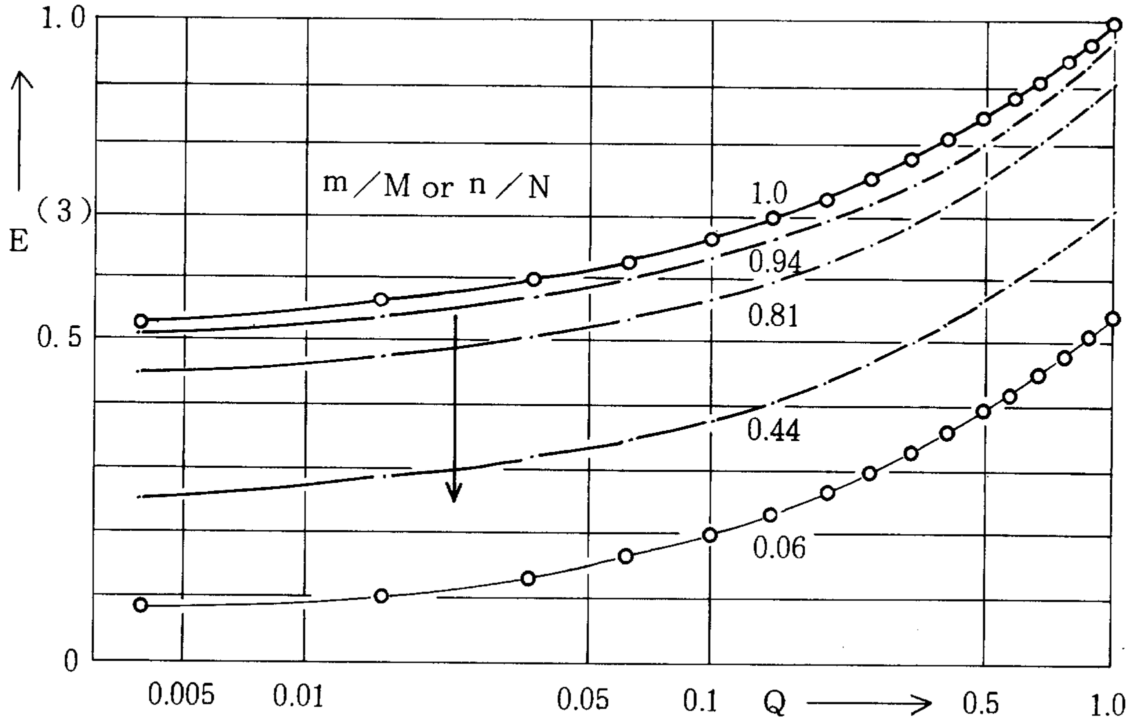


Fig. 5 An example of $E^{(3)}$ (Q)——

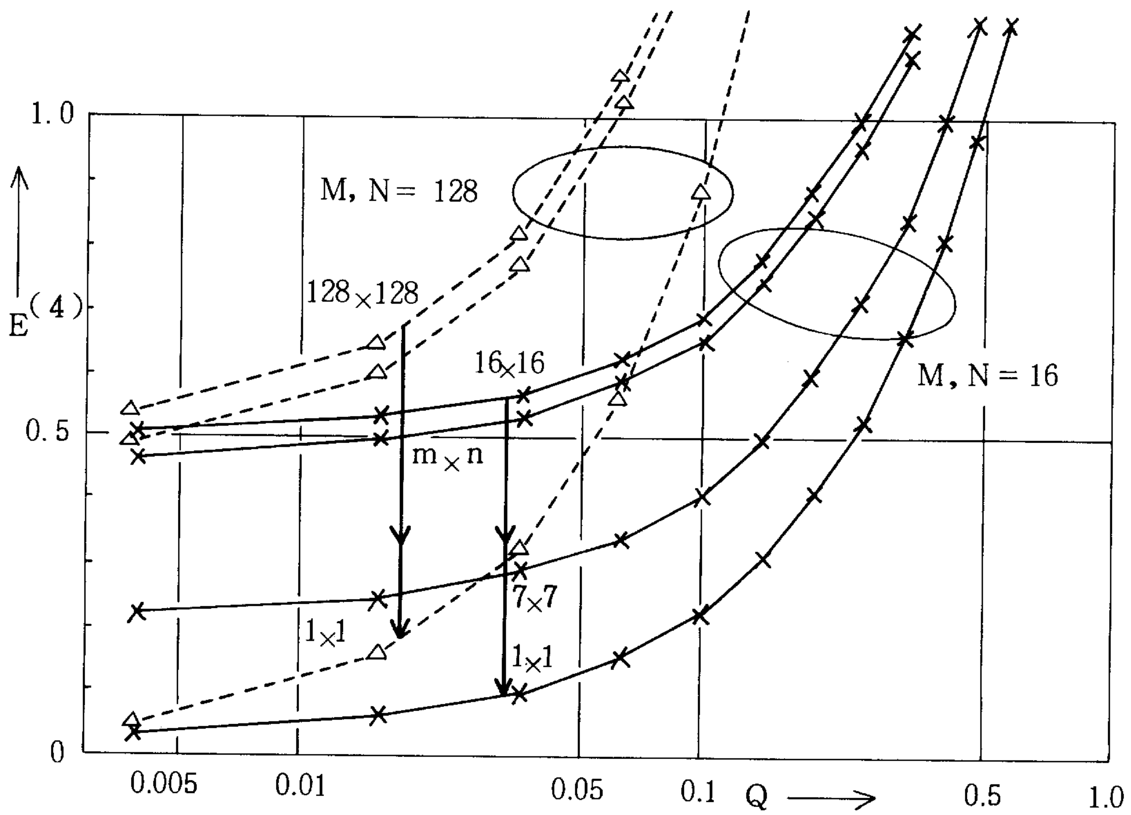


Fig. 6 An example of $E^{(4)}$ (Q)——mixed-method I ——

characteristics of $E^{(k)}(Q)$ measures for generalized FFT - method (k=3) and generalized mixed - method I (k=4) are shown. If the strict value of $E^{(k)}(Q)$ is needed, it is numerically obtained from Table 2. Each $E^{(k)}(Q)$ has the tendency of the rapid decreasing depending on the decrease of cs - ratio Q.[9] The next properties of $E^{(k)}(Q)$ are known from these results.

① $E^{(k)}(Q)$ for FFT - method (k = 3) depends only on Q, m / M and n / N. Others (k=2, 4, 5) depend on M, N, Q, m / M and n / N.

② E - measures $E^{(k)}(Q)$ for CS - methods never fall below 0.5, and those for generalized ones fall below 0.5.

③ Computation costs of CS - methods can be reduced to 50% of those of 2 - D FFT method, and those of generalized ones be reduced to 100%.

3. 3 Relations among CS - methods

It is important to find out a procedure to choose the best method among the generalized CS - methods for arbitrary value of Q.

In Fig. 7, $E^{(k)}(Q)$ measures for k = 2, 3, 4 and 5 are depicted together. There exists a crossing point Q_0 of them for a given pair of M and N, as shown in Fig. 7, and Q_0 is formally derived by letting $E^{(2)}(Q_0)=E^{(3)}(Q_0)=E^{(4)}(Q_0)=E^{(5)}(Q_0)$.

Q_0 is given by eq. (17) and is called "*critical point*" of CS - methods.

$$Q_0=(\log(M+N))/(M\times N) \tag{17}$$

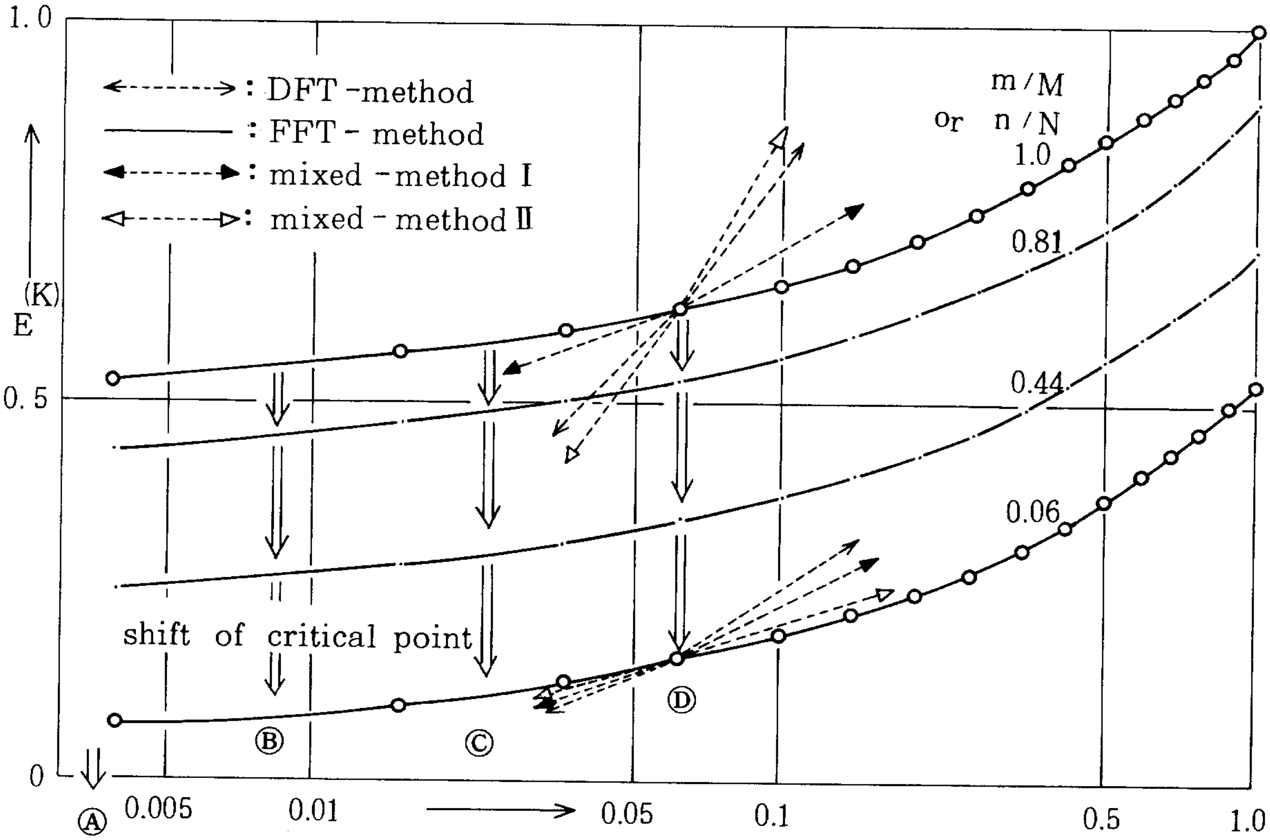
The critical point has the following properties:

① Only one critical point exists for a pair of M and N.

② As any one of the CS - methods requires the same computation cost for $Q = Q_0$, any one of them can be equally chosen at this point.

③ The order of $E^{(k)}(Q)$, k =2, 3, 4 and 5 where $Q\geq Q_0$ is just the reverse order of them where $Q<Q_0$.

④ While Q_0 is independent w. r. t. m and n, $E^{(k)}(Q)$ varies with m and n.



Ⓐ, Ⓑ, Ⓒ, and Ⓓ are the critical points for M, N= 128 , 64, 32 and 16, respectivily.

Fig. 7 Examples of $E^{(k)}$ (Q), k = 2, 3, 4, 5 and their "critical point"

Table 5 The order of priority to choose the best method

partitions of Q (%)	M, N=16 mn=15 (%)	M, N=128 mn=120(%)	CS-methods	generalized CS-methods
$0.0 \sim Q_0$	0.0~6.3	0.0~0.3	A → D → C → B	A → D → C → B
$Q_0 \sim Q_2$	6.3~14.5	0.3~1.1	B → C → D → A	B → C, D → A
$Q_2 \sim Q_5$	14.5~18.0	1.1~1.2	B → C → D	B → C, D
$Q_5 \sim Q_4$	18.0~26.0	1.2~5.9	B → C	B → C, D
$Q_4 \sim 100.0\ (Q_3)$	26.0~100.0	5.9~100.0	B	B

A: direct-method, B: FFT-method, C, D: mixed-method, I, II

: The order of priority varies with parameters m and n.

Using the critcal points Q_0, together with the permissible points Q_2, Q_3, Q_4, and Q_5, the order of priority to choose the best method is determined as Table 5. Some numerical examples are also appended in Table 5. As these results are applicable for any combinations of M, N, m and n, we can usually use this table to choose the best one among the generalized CS-methods on the real occasion.

4 Other considerations

Properties of the CS-methods and their generalized ones and the procedures to use these methods were made clear in section 3. Other related considerations are summarized in this section.

4.1 Extension to inverse 2-D DFT and other transforms

As two-dimensional inverse DFT (2-D IDFT) of $\mathbf{F} = \{F_{kl}\}$ defined by eq. (18) is equivalent to 2-D DFT of $\mathbf{F'} = \{F_{kl} / (M \times N)\}$ defined by eq. (19), CS-methods can be easily extended to "element selective methods" (*ES-methods*) for calculating 2-D IDFT. Figure 8 shows the procedure to realize the ES-methods by using CS-methods. All results concerning the CS-methods given in section 3 are valid for ES-methods without any modification.

$$f_{ij} = \frac{1}{M \times N} \sum_{l=0}^{N-1} \sum_{k=0}^{M-1} F_{kl} \cdot \exp\left\{+2\pi J\left(\frac{ki}{M} + \frac{lj}{N}\right)\right\} \quad (18)$$

$$f_{-i-j} = \sum_{l=0}^{N-1} \sum_{k=0}^{M-1} (F_{kl} / (M \times N)) \cdot \exp\left\{-2\pi J\left(\frac{ki}{M} + \frac{lj}{N}\right)\right\} \quad (19)$$

Although the discussions are restricted only to Fourier Transform in this paper, CS-methods are easily extended to other orthogonal transforms such as Walsh-Hadamard Transform (WHT), which are also important in image data processing.

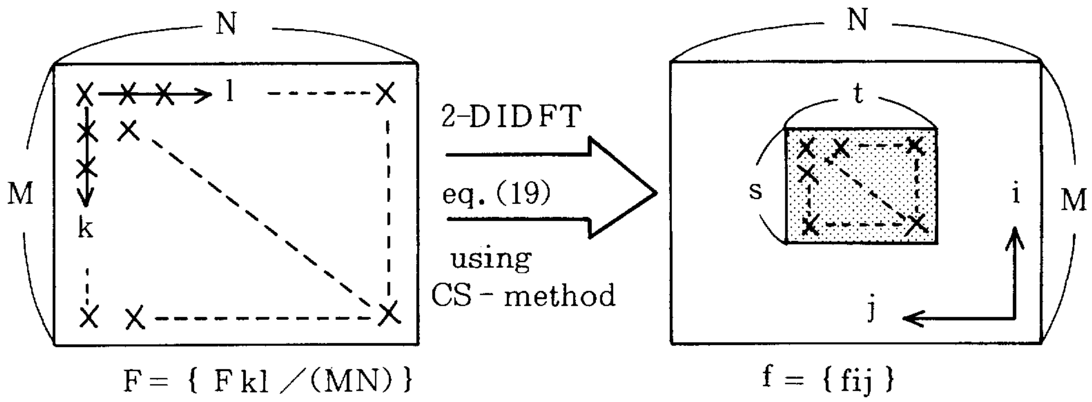
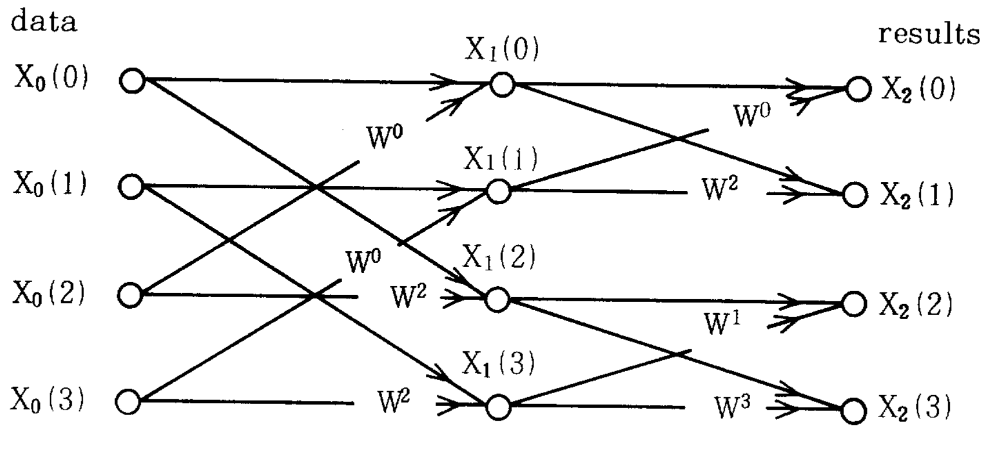


Fig. 8　ES-methods for calculating 2-D IDFT by CS-methods



W= exp{-2πJ/N}, J=√-1

Fig. 9 Signal flow graph for FFW (N = 4, base = 2)

Figure 9 is a signal flow gragh of FFT (N=4, base=2), where an example of batterfly operations is denoted by eq. (20).

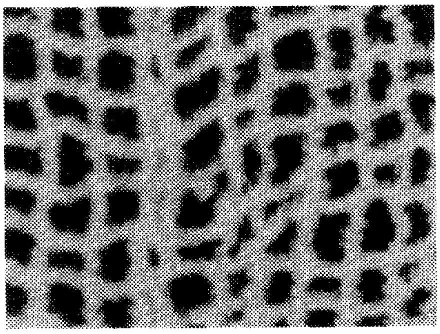
$$x_1(2)=x_0(0) + W^2 . x_0(2), \quad W=\exp(-2\pi J / N) \qquad (20)$$

As W is always equal to 1 in WHT, CS-methods proposed for 2-D DFT are applicable directly to 2-D WHT when we exchange the number of multiplications in FFT with that of additions in WHT. Therefore, the results obtained in section 3 are valid also for WHT and also for other orthogonal transforms.

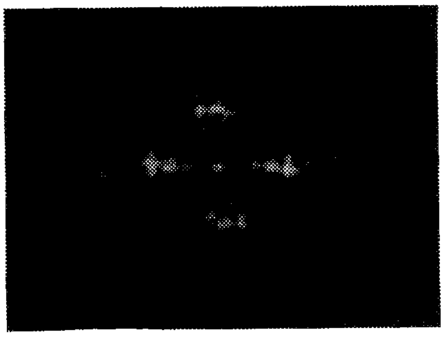
4. 2 Applications

CS-methods play an important role for extracting the specified spectrum features of images as quickly as possible.

Figure 10 (a) is an image data of chemical fiber fabrics where a "float" defect exists. (sampling pitch = 0.05mm×0.05mm, density levels = 128 and $M\times N$ = 64×64) It is important in fabric image processing to inspect the periodicity of warp or weft in order to recognize warp or weft defects.[10] In this case, it is enough to calculate only a subset consisted of the neighboring components on the corresponding frequency axis. Figure 10 (b) shows Fourier spectrum of Fig. 10 (a) and (c) shows two subsets of (b), which are calculated by means of mixed-method I (where $s\times t$ = 10×40 and 40×10). These results were used for the inspection of the

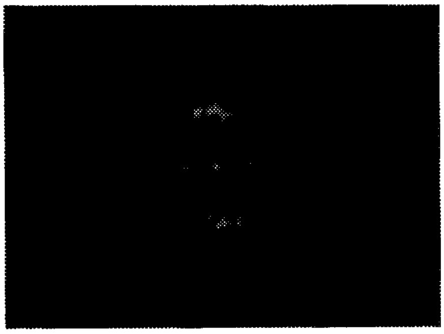


"float defect"
(chemical fiber fabrics)
(a) an image data



(b) Fourier spectrum
of (a) celculated by 2-D FFT
($M_x N = 64 \times 64$)

$s \times t = 10 \times 40$
(for inspecting warp defect)



$s \times t = 40 \times 10$
(for inspecting weft defect)

(c) two subsets of (b) calculated by
mixed-method I ($M \times N = 64 \times 64$)

Fig. 10 An application of CS - method for extracting the specified spectrum features

periodicity of warp and weft.

The processing time needed in the above experiment is summarized in Table 6, where other experimental results are also shown. The programs for experiments are written in FORTRAN (just below JIS 7000) and executed by minicomputer (PFU - 200 system). It is known that these experimental results are well coincident with the theoretical ones given in section 3.

For example, theoretical boundaries given in Table 3 are well

Table 6 Experimental results of calculation time

s × t		size of image data M × N	
		16 × 6	32 × 32
9		978 ms	5004 ms
25		1218	5486
49	(64)	1578	6202
81		2062	7170
121	(160)		8370
169			9841
2-D FFT		1684 ms	9490 ms

▷theoretical boundaries given in Table 3

coincident with experimental ones shown in Table 6, where $\longrightarrow$ and ——▷mean the theoretical boundaries.

CS-methods and their generalized ones are also important for realizing ideal low-pass or high-pass filtering of images as quickly as possible. If the low-pass band consists of s×t components, the number of multiplications required for realizing this filtering is evaluated as follows:

using 2-D FFT method

$$\{(M\times N)\log(M\times N)\} + [(M\times N)\log(M\times N)] \qquad (21)$$

using CS-method

$$\{(M\times N)\log M + s.\ N\log N\} + [t.\ M\log N + (M\times N)\log N] \qquad (22)$$

, where { } is the number of multiplications for Fourier transform and [ ] for inverse Fourier transform.

Furthermore, as two-dimensional filters in image processing are usually designed as FIR filters, such as 5×5 smoothing and 9×9 Laplacian filters where the majority of elements in the outside of the mask are equal to zero, generalized CS-methods offer a powerful aid especially for fast convolution processing using these FIR filters.

5 Conclusion

This paper has established intermediate methods between 2－D DFT and 2－D FFT methods for calculating only a subset of spectrum components of images. These methods are really utilized in several applications as shown in section 4. 2, and are easily extended to inverse transforms and other orthgonal transforms as given in section 4. 1. Several utility restrictions against these methods and the procedures to use them were strictly made clear in section 3.

In order to make these methods more efficient for several applications, it is important to put the following points under investigation.

(1) As the savable computation cost due to these methods in about 50％ at most (in the not generalized cases), some effective countermeasure must be taken to improve the component selective Fourier calculations.

(2) From this viewpoint, it would be prospective to develop "a component selective method for calculating 1－dimensional FFT".

(3) Implementation of these methods as a subroutine library and improvement of them in combination with Onoe's method[7] are also important for the real applications.

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