The Laboratory may be more Effective than the Lecture

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"There's a terrible problem that I run into in teaching, which is that when you tell people something, you keep them from knowing it. If they find it on their own, they'll know it in a way they never will if you tell them. What I try to more and more is to bring students to my studio and get them really working."

...Richard Benson

"Personally, I am always ready to learn, although I do not always like being taught."

...Winston Churchill

This paper argues that the lecture method is not well suited to gaining an understanding of mathematics or related fields of application. Human beings must be actively involved if they are to truly learn. Lectures are not conducive to this, indeed, they are anti-conducive. Lectures in many course might best be replaced in large part, or even completely, by a laboratory method. This could mean case work in groups, use of a computer with suitable software, or working in a group with a computer and suitable software. A caveat: when I speak of computer software for learning, I do not mean CAI drill courseware. Rather, software which frees the student from calculational drudgery that would otherwise conceal the important concepts, which allows methods to be used which would not be feasible by hand, which allows visualization and animation, and which enables the student to gain experience with a variety of (more) realistic examples, instead of one or a few highly simplified and idealized examples.

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1 Almost all quotes heading sections are taken, with gratitude, from the extensive collection in Davis, Porta, and Uhl’s electronic textbook “Calculus&Mathematics”. Such quotes (i.e. from Davis et. al) are attributed only to their originator. Quotes from other sources are cited in [citation number] format as well.
Mathematical Ghetto

Even though I will use the term "mathematics" to describe the field of study I write about, I really mean something more general than what likely comes to the reader's mind when reading the term. I include mathematics in a wide meaning, beyond just what is done and taught by professional mathematicians. I include all fields of activity which are heavily mathematical in their nature, regardless of the job title of the person doing it.

In North America, within most curricula, mathematics has been so thoroughly dissociated from all other subjects that students generally encounter little evidence [to counter the assumption that] ... mathematics is a highly esoteric subject which, despite having some practical applications, possesses only tenuous connections to other areas..." [1]. Part of the reason for this can be reasonably attributed to the limitations imposed by the lecture format, as well as the emphasis on a logical, deductive development of the subject.

In Japan

Japan prides itself on having the highest overall results on school mathematics exams in the world. The amount of attention paid to mathematics education, even to the casual observer, clearly exceeds that in the United States, Canada, or Australia. Whenever I am in a bookstore in Japan (other than the ubiquitous tiny, corner bookstores), I am always struck by the great number of books on mathematical, scientific or generally technical topics. These include technical books for the specialist, study books for the student, and books aimed at the curious layman. The sections on management include dozens of technical titles on operations research, statistics, quality control and other topics assuming a mathematical background. When visiting comparable (or even more than comparable) bookstores in the United States, Canada, or Australia, I see far fewer technical or mathematical titles overall, and the business/management section typically includes no books presupposing any mathematical knowledge.

As an example, at a typical branch of one of the major bookstore chains near to my university, there were 209 different titles of study books for high school mathematics. (That is two hundred and nine different titles of books, all devoted to drilling, strengthening, etc. topics in high school mathematics.) This was only one side of one island among several devoted to study books for various topics. Education in general, and mathematics education in particular, is taken seriously in Japan.

On the other hand, the growing numbers of students who dislike, and increasingly, avoid mathematics courses is a matter of growing public concern among mathematics teachers in Japan [2-7]. In the September, 1994 issue of the
The Laboratory may be more Effective than the Lecture

Journal of the Japan Society of Mathematical Education the heads of the four primary societies in Japan concerned with mathematics and mathematics education declare a crisis in mathematics education in Japan [2]. In their fifth point, they call for the use of the computer to promote active, independent work by students. The vice chairman of the Japan Society of Mathematical Education, in a preface to the November 1994 issue of the societies journal uses the word "math-leaving" (「数学離れ」) six times in four paragraphs [3]. The perceived crisis is a common preface to symposium talks on curriculum reform. It is a common topic of discussion at meetings of high school and university mathematics teachers2.

However, in a survey conducted by the Japan Society of Mathematics Education, [6] the expressed interest in using computers or graphing calculators in high school mathematics instruction was extremely low (10% and 16% respectively). This is in contrast to the interest in calculators of some sort in elementary and middle schools, which exceeded fifty percent. Another noticeable result was the decided lack of interest in what other countries are doing, or in connections to other fields. This survey was sent to 980 selected teachers in influential positions at national universities, at middle and high schools attached to national universities, and members of the society 347 replies were received, so it is unclear how representative it is of the majority of teachers "in the trenches".

The State of the Students

"[The normal student attempts to fix formal terminology] in his memory because it means nothing to his intelligence."

...Blaise Pascal

"Knowing the formal definitions of a bunch of words is no guarantee that one can actually express anything comprehensible."

...James O. Bullock [1]

Despite having students who are able to score well on standard exams, teachers and education researchers report that these same students do not do well when asked to demonstrate understanding of what they have supposedly learned. Nakazawa (Nakazawa 1995) reports "After passing a university course in matrices, the students had no idea what a matrix was or what it could be used for. They knew only how to manipulate the symbols." When I did some testing with a class of third year management students, I found that the students could easily perform the

2 Here I cite the above references in general, numerous personal conversations with high school and university mathematics teachers, and university teachers from other fields.
algebra of finding an inverse to a linear function when asked. But, fewer than ten percent understood in the slightest what an inverse function is [8]. These included several students who had taken a year of calculus in high school, and did quite well on standard problems involving equations of lines and taking derivatives.

In the United States the reported numbers of students who fail or withdraw from calculus in university is reported as "at least 60%" [9] and as "50% on average by Steen [10] In Japan, the problem appears to be more one of avoidance of enrolling in the course in the first place, and so figures are more difficult to estimate. However, as written in the lead paragraph above, it is clear from the literature, as well as anecdotal conversations with high school and university instructors at meetings and conferences, that there is a sense of crisis (among at least some educators) in Japan as well.

Let us return to the question of what is known by those students who do take and pass a course in calculus. David A. Smith and Lawrence C. Moore of Duke University, originators of the Project Calc curriculum say:

"What our students have learned from school mathematics - more precisely, what they have invented for themselves - is a set of 'coping skills' for getting past the next assignment, the next quiz, the next exam. When their coping skills fail them, they invent new ones. The new ones don't have to be consistent with the old ones; the challenge is to guess right among the available options and not get faked out by the teachers tricky questions. At Duke, we see some of the 'best' students in the country; what makes them best is that their coping skills have worked better than most for getting them past the various testing barriers by which we sort students. We can assure you that this does not mean that our students have any real advantage in terms of understanding mathematics." [11]

Tall reports the case of Sawyer. Sawyer was teaching a course in functional analysis. by referring to theorems of real analysis. Since the students had just finished a course in real analysis, he expected they should know these theorems. However, they did not remember them at all. As Tall recounts it, they did not remember ever hearing about them. "The reason was that in their university lectures, they had been given formal lectures that had not conveyed any intuitive feeling; they had passed their examination by last minute revision and rote." [12, p 16]

Zorn reports the following, quite charming, comment by a student [13]:

Algebraic symbols are what you use when you don't know what you are talking about.
The Laboratory may be more Effective than the Lecture

How many instructors who read this comment, or the anecdote before it, will nod their heads, smile wryly, and say “Yes, that is so common”? This is certainly what happens in either Japan or the US when the author relates these stories. Yet, is that not to give up and accept the situation as irreparable? It is the plea of this article that we can do something about this situation. A great deal of its cause is rooted in the way we go about attempting to teach mathematics. The fault, I believe is in large part rooted in the use of the lecture method.

The Problem with Lectures

"Is it life, I ask, is it even prudence,
To bore thyself and bore the students?"

... Johann Wolfgang von Goethe

"Can one ever understand a theory if one builds it up right from the start in the definitive form that rigorous logic imposes? No; one does not really understand it ...one retains it only by learning it by heart."

... Henri Poincare

The National Research Council of the United States (NRC) in its report "Everybody Counts" says:

"Students simply do not retain for long what they learn by imitation from lectures, worksheets, or routine homework. Presentation and repetition help students do well on standardized tests and lower-order skills, but they are generally ineffective as teaching strategies for long-term learning for higher-order thinking, and for versatile problem-solving." [14]

The results of research in cognitive science over the last many years, as well as in mathematics education, are strongly against lectures as a method of learning suited to the human mind [15-19]. According to research in cognitive science, learning requires active involvement on the part of the learner. This involvement is mental of course, it does not have to be physical. But the involvement must be there. Lectures have been described as a method of transferring the contents of the notes of the professor to the notes of the student without passing through the mind of either [17]. The fact appears to be that listening to lectures, even if notes are taken, does not cause students to learn mathematics. Further, working through standard problem sets does not cause the student to learn
more than how to do just that. He or she does not learn how to use the techniques they are drilling [18, 20-22].


"It is widely recognized that lectures place students in a passive role, failing to engage them in their own learning. Even students who survive such courses often absorb a very misleading impression of mathematics -- as a collection of skills with no connection to critical reasoning."

Engagement is what appears to be the most important factor in human learning. The learner must be actively engaged in the process. Further, this involvement must be based in the learners own experience. The normal method seen in a typical high school textbook is to introduce the formalism first. This is in line with a long tradition in mathematics. But, trying to sequentially develop formalisms, before the student has a broad range of experience, can lead to false generalizations [24]

In a traditional mathematics lecture, the teacher attempts to explain everything as clearly as is possible. He will often use pictures to make certain concepts intuitively clear. The definitions and ideas will be presented in a logical sequence. However, according to research in cognitive psychology, "... the brain was designed by evolution to deal with natural complexity, not neat 'logical simplicities'... " Hart 1974, page 76, cited in [25, 26]. In a prior section of the book, he makes the statement. [26]

"We have all been brainwashed by the undeserved respect given to Greek-type sequential logic. Almost automatically curriculum builders and teachers try to devise logical methods of instruction, assuming logical planning, ordering, and presentation of content matter ... They may have trouble conceiving alternative approaches that do not go step-by-step down a linear progression ... It can be stated flatly, however, that the human brain is not organized or designed for linear, one-path thought."[emphasis added] ( ibid, page 52)

There is no concept, no fact in education, more directly subtle than this: the brain is by natures design, an amazingly subtle and sensitive pattern-detecting apparatus." ( ibid,page 60)
The Laboratory may be more Effective than the Lecture

Human are also social animals. Both cognitive and educational research has shown that working with others promotes learning [9, 15-18, 24, 27-29]. Liebman proposes the reform of Operations Research education to scale down the role of lectures. She urges small group work to involve students. "We frequently offer lecture based courses because lecturing is the way in which we were taught, because lecturing is the only way we know how to teach, because lecturing enables us to cover many topics, and because lecturing provides us an environment in which we feel in control. However, if our objective is to maximize learning, then the use of lectures alone is inadequate." [17]

In order for knowledge to be learned and retained, cognitive processing must take place. Voss relates "Schoenfeld (1987, 1988, 1991) has stressed that children need to use mathematics as a tool for recognizing and solving problems, instead of trying to find the answer as quickly as possible. Schoenfeld (1988) has noted that traditional instruction does not accomplish this goal even when students learn the course contents. [emphasis added] Schoenfeld also discusses the importance of metacognition and social factors in mathematics instruction, that is, how knowledge of one's own thought processes and the use of self-monitoring procedures as well as participation in small groups facilitate performance Schoenfeld (1987)." [15] In the area of research into physics education, Voss goes on to report work by Chi et.al.(1989), who "found that compared to poor learners good learners in solving physics problems explain each step to themselves; refine, elaborate, and evaluate conditions needed to take a step in the solution process..." [15]

"When students must learn complex and technically difficult material, active learning is much more effective than passive learning" [17]. Cognitive psychology researchers believe that cognitive rehearsal is necessary, where material is restructured in the mind, before effective learning can take place. This sort of rehearsal just does not take place when students are passive observers in a lecture, not even if they take notes.

Of course, even if students do take notes, they all too commonly cannot do test questions based on those notes [30]. But, even assuming they can do the test questions, they typically do not retain useful knowledge to take into later course, or their life work [14-16, 18, 20, 22, 31-36]. It is just this problem which is the most serious, and probably uncorrectable with the lecture system. The knowledge that students will find useful goes beyond predigested tidbits suitable for test questions. "In active learning and peer-grouped learning, students form life-long learning skills, develop the ability to explain complex material, learn how to work productively with others, and are more likely to develop effective group leadership skills.... If our educational objective is to increase student learning, then we should reduce our reliance on lectures and provide a more diverse and enriched learning environment." [17]

Sfard develops a three stage theory of cognition, involving what she terms interiorization, condensation, and reification. By interiorization she means that some new process is performed on objects with which the learner is
already familiar. Then, the new concept is seen to bring these objects together into a new whole. This is 'condensation'. Finally, the learner learns how to treat this new whole as a single object, a thing. This is 'reification'. The learner now has acquired a new abstract object. She emphasizes that this must proceed from the concrete, the familiar, the meaningful [20].

... where do these abstract entities come from .. ? Whoever looks into a standard mathematical text may be tempted to give the simplistic answer: abstract objects are nothing but the products of (formal) definitions. Such an answer, however, would not be just simple -- it would be simplistic. Even if mathematicians feel that theirs is the godlike power of bringing abstract objects into existence just by saying "Let it be," the act of creation may, in fact, be not as straightforward as that. ... Reconstructing the process of construction is never easy. 'structural conceptions develop usually out of operational;' or, in other words, 'abstract objects emerge from certain computational processes.' (Sfard 1992, page 61)

".. The learner must have a lot of determination, stamina, and intellectual discipline to cope with the demanding task. Because of the inherent difficulty of reification, nothing will happen without a genuine drive for understanding. .. the teacher cannot just put the abstract objects into the students' minds --- he or she will not be able to do for the learners what the learners do not want to do for themselves. (ibid, page 84)

According to Oliver Sacks "... learning involves involves the construction of complex percepts - syntheses of representations from every part of the cerebral cortex - brought together into a contextual unity, or 'scene'.' [37, p 53] Notice that Sacks says these percepts are "complex", they involve large areas of the brain, and that they must come together into a "contextual unity". This unity must be in the context of the learners experience, of course. Sacks goes on to say that what is synthesized is not yet learned: "Such syntheses can be held together for only a minute or two - the limit of short term memory - and after this will be lost unless they can be shunted into long term memory." [37, p 54] According to Bartlett, "Several studies [have shown that] lectures are a relatively poor way to promote learning. ... even the best medical students [attention] peak[s] at around 15 minutes of the lecture." [31] If we combine these last two observations, it does not bode well for learning in a lecture environment. One can easily visualize the contents of the lecture remaining in short term memory only long enough to be transferred to student notes, and
The Laboratory may be more Effective than the Lecture

then, with the possible exception of the fifteen minutes of the lecture, evaporating. A human cannot long, if at all, maintain an active mental state when passively listening to a lecture.

"Consider the assertion ... that 'Language is a surrogate for experience.' If this were not so, it would be hard to understand how language could be used to inform us about events and objects with which we had no direct contact. Nonetheless, language can be distinctly inferior to experience. Our perceptual apparatus delivers (usually) veridical, organized detailed representations. Language may not do any of that. In particular, the structure of language is particularly ill-suited for communicating the spatial organization to which our perceptual apparatus seems tuned." [16].

Bullock compares formalism to memorizing vocabulary. We can memorize the definitions and spellings of many words. This is not entirely useless. But, by itself, vastly insufficient. The point of language is to be able to express clear thoughts. When writing, through the construction of sentences and paragraphs "... knowing the formal definitions of a bunch of words is no guarantee that one can actually express anything comprehensible. One must obtain practice in expressing complete thoughts with language. In mathematics, this means constructing examples which deal with ... applications." [1] Whereas, in fact, we require of our students laundry lists of definitions and standard equations (quadratic equation, several ways to factor the cubic, logarithms, etc. etc) that are not unlike asking the students to commit a dictionary to memory (Bullock's comparison).

If mathematics, or its fields of application such as operations research or data analysis, consists of constructing examples which deal with applications, then the spoken language would seem to be an inadequate means of communicating how to do this.

**The Problem with Clear Explanations**

"There is a real hypocrisy, quite frequent in the teaching of mathematics. The teacher takes verbal precautions, which are valid in the sense he gives them, but that the students most assuredly will not understand the same way."

... Henri Lebesgue

"...concepts which expert mathematicians regard as intuitive, are not intuitive to students..."

... David Tall

"It is essential that abstractions come only after the students have experienced what they are learning about."

... Bill G. Aldridge [38]
CHUKYO KEIEIKENKYU

The trouble with "clear" explanations is simply that what is clear to the teacher is often not at all clear to the student. Even worse, as is all too often true, what the student sees as being "clear" is not the same as what the teacher sees.

We as teachers have spent so many years subsuming these ideas into our consciousness that they become second nature. We can forget entirely how unobvious they are to most beginners. In fact, they may have been unobvious to us when we were students. How often do we not understand until we have to teach it the first time?

Even when we draw pictures that seem to us to have only one interpretation, there may be other interpretations that our long training make us blind to. For example, consider Figure 1, at right. If you saw only the middle figure, you might perceive the plug-ugly face (call him 'beast'), or you might perceive the nude (call her 'beauty'). But, whichever interpretation you perceived in the middle figure, would you even be aware that the other existed? Once you see the series of three, you can now perceive either 'beast' or 'beauty'. Had you seen only the left or the right figure, it is a near certainty you would never guess there is more than one view of what the drawing portrays. (Reproduced from 42)

How often does this happen in mathematics classes? We probably can never know entirely. The only check we have is that students answer test questions correctly. But, do test questions actually test true understanding. If they did, we would expect students to retain this understanding and be able to apply it in the months and years after they leave the course. We would also expect them to be able to handle questions not like examples done in class. We have seen that this is not the case.

"... [Concepts] which expert mathematicians regard as intuitive, are not intuitive to students. ..." Intuition depends on the students past experience. It depends on the "cognitive structure" of that individual. In particular, for mathematical concepts where the instructors and the students experiences are (we hope!) different, we cannot expect intuitive understanding of diagrams or arguments to be the same. [Tall, 1991 #109].
The Laboratory may be more Effective than the Lecture

Tall recounts the results of a study by Orton where 110 calculus students were asked to explain in detail the meaning of the figure similar to Figure 2, shown at right [25]. Students were individually interviewed. "43 students seemed incapable, even when strongly prompted, to see that the process led to the tangent of the curve. ... they appeared to focus their attention only on the chord PQ, even though the diagram and explanation were intended to insure that this did not happen. Typical unsatisfactory responses included "the line gets shorter", "it becomes a point", and "the area gets smaller".

Although the students in the example above understood the diagram in an unexpected sense, might this not at least be detected upon examination? Perhaps, but comprehensive examinations in lecture courses normally come far after the fact. Even if discovered by student answers to test questions, by the time the test is given, the student may have waded through considerable material not understanding it. The student may not even have realized the lack of understanding!

Worse yet, the student might answer the test question correctly, but for the wrong reasons. Consider the following study by Tall [25]. Nine 16 year old calculus students in England were individually interviewed. They were asked if the following statement is true: "In Figure 3, as B→A the line through AB tends to the tangent AT." To a mathematics instructor, this certainly seems like an unambiguous statement. But, in fact, of the students who said the statement was true, four "linked the symbol B→A to vector notation and visualized B moving to A along the line BA. For them the line segment BA 'tends to' the tangent, but in a completely unexpected sense." A further caution in this tale is that one of the students said the statement was false because "way off at infinity, the line AB and the tangent T would always be always be far apart, no matter how close A and B become." Even though we can debate about what it means to be "at infinity", this answer undoubtedly shows more understanding than many (if not all) of the correct responses.

Intuition is a global resonance in the brain and it depends on the cognitive structure of the individual, which in
turn is dependent on the persons previous experience. There is no reason at all to suppose that the novice will have the same intuitions as the expert, even when considering apparently simple visual insights. Mathematical education research shows that student's ideas of many concepts are not what might be expected. [25]

Freudenthal put it this way, "One finally masters the activity so perfectly that the question of how and why [students don't understand them] is not asked anymore, cannot be asked anymore and is not even understood any more as a meaningful and relevant question." [21]

The Mathematics We Teach

"Elementary mathematics . . . must be purged of every element which can only be justified by reference to a more prolonged course of study. There can be nothing more destructive of true education than to spend long hours in the acquirement of ideas and methods that lead nowhere. . . . [The] elements of mathematics should be treated as the study of fundamental ideas, the importance of which the student can immediately appreciate; . . . every proposition and method which cannot pass this test, however important for a more advanced study, should be ruthlessly cut out."

. . . Alfred North Whitehead

"We no longer ask students to understand. Now it is [algebraic] manipulation pure and simple. . . . What we're teaching is not only the wrong thing- in that it is not what students will use [outside the calculus classroom]- it is obsolete [because of computers and calculators]. It is like spending all your time in elementary school adding and subtracting and never being told what addition and subtraction are for."

. . . Ronald Douglas,

Committee on the Mathematical Sciences in the year 2000

The manner in which mathematics is presented in the high schools is strongly influenced by the existence of university entrance examinations3. A look at the table of contents of a high school text book, will show a large number of topics covered in the school year, some surprisingly advanced. (E.g. The Cayley-Hamilton Theorem in the second year of high school. The presence of many of these topics appears to be motivated by a desire to cover

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3 Numerous personal conversations with high school and university mathematics teachers, and university teachers from other fields. Common sense.
The Laboratory may be more Effective than the Lecture
topics that appear on the entrance examinations of prestigious universities.)

The presentation, as given in the text book, is typically formal definitions, followed by standard formulas, followed by standard example problems, followed by practice problems modeled on the problems.

The French mathematician Henri Lebesgue considered the problem of teaching at great length during the first third of this century.

Unfortunately, competitive examinations often encourage one to practice this little bit of deception. The teachers must train their students to answer little fragmentary questions quite well, and they give them model answers that are often veritable masterpieces and that leave no room for criticism. To achieve this, the teachers isolate each question from the whole of mathematics and create for this question alone a perfect language without bothering about its relationships to other questions. Mathematics is no longer a monument but a heap.

... Henri Lebesgue

Some Existing Courses Based on the Computer

"Mathematics-speaking machines are about to sweep the campuses... The ready availability of powerful computers will enable students to set new ground rules for college mathematics... Teachers will be forced to change their approach and their assignments. They will no longer be able to teach as they were taught in the pencil and paper era...

... Lynn Arthur Steen,

... Past President of the Mathematical Association of America

There are a number of courses in first and second year university mathematics being developed in North America that center on the computer. One of the earliest, and possibly the most radical, is the "Calculus & Mathematica" course at the University of Illinois and Ohio State University. These are both large research universities with very large programs in undergraduate mathematics education for very many fields of study.

This "Calculus & Mathematica" course uses the computer algebra system (CAS) "Mathematica" and has been
under development for over five years. It is beyond the experimental stage. There are no lectures. The textbook which the student buys is completely contained on floppy disk. (Although, separate printed editions, and booklets can be purchased.) There are sets of "literacy sheets" that must be worked by hand, to insure that the student must work by hand. But, otherwise, the entire course is done "on the computer; on screen". The material is not rote drill, but sets of tutorials, problems, and open-ended questions for students to work through with their own intelligence, ingenuity, and perseverance. Students are strongly urged to work in groups.

Here is a quote from the post-course student evaluations:

"Calculus & Mathematica is the only way I've seen to teach any math higher than geometry in such a fashion that students gain a fundamental understanding of the concepts involved. In a normal class, the opportunity to discuss how and why mathematics works gets lost in all the drudgery and rote calculations. Mathematica takes care of the drudge calculations and leaves the student free to concern himself primarily with the essential concepts that truly make up math." [39]

This student has eloquently put the heart of the case for basing a mathematics course in the laboratory, and not the lecture hall. She is actively involved with her learning process. She can visually (and algebraically) initiate the mathematics she investigates. Working in a group, she is working in the same way she will in her future workplace, when she and her coworkers use computers to perform an analysis, or complete a project.

In the electronic version of the textbook, the authors of Calculus & Mathematica (C & MMA) provide an extensive discussion of their experience, including rebuttals of objections to such extensive use of computers. The University of Illinois School of Education has performed a controlled study to measure the performance of students of C & MMA. The results are summarized as follows:

"1) Calculus & Mathematica students are more likely to try multiple approaches to a problem than students from the traditional course

"2) Calculus & Mathematica students cannot be distinguished from students in the traditional course on the basis of their ability at hand calculations.

"3) Calculus & Mathematica students demonstrate a richer ability to identify relations between calculus ideas than students in the traditional courses
The Laboratory may be more Effective than the Lecture

"Additional fillip studies of Calculus & Mathematica students done by the University of Illinois indicate that Calculus & Mathematica students are decidedly more likely to excel in post-calculus science and mathematics than their statistical twins from the traditional courses."

One of the most important factors is long-term retention of knowledge. We have seen above, that long-term retention by students is notoriously poor in the case of lectures. The results of the C&MMA studies do show much better retention, through the better performance in subsequent classes. The developers attribute this to the active involvement, and explicit construction of mathematical objects by the students working in small groups directly with the Mathematica at the computer.

Other courses involving independent or semi-independent use of CAS systems by students are becoming too numerous to keep track of. The university of Iowa (Stroyan), Project CALC at Duke University (mentioned above), and the programs developed at Rose-Hulman Institute of Technology being among the best known.

Another approach is that of Dubinsky, and his group, at Purdue [22, 33]. They have written a special program for performing mathematics called ISETL. It is algebraic, and not visual. The students are presented with mathematical ideas that they must program into ISETL. Or, they are given theorems that they must establish as true or false by fashioning and testing ISETL programs. The developers stress the need for cognitive reflection in learning mathematics.

On the surface, this appears best suited to developing the ability to prove mathematical theorems, as opposed to fields of application mathematics. However, the first textbook written to use ISETL, is for the study of discrete mathematics, certainly a field with applications in many areas.

"We reject the idea that people learn mathematics spontaneously by listening or watching while it is being presented. We do not feel that mathematics is learned effectively by working with many examples and trying to extract their essential features." [22, p 47]

"Because of these beliefs, we do not choose a teaching methodology that is restricted to lectures, showing applications, having students work on set problems, and testing their performance on examinations." [22, p 48]
CHUKYO KEIEIKENKYU

"What the student does in order to learn is much more important than what the professor does in order to teach. ... A powerful effect of computers for most students is to keep them deeply involved. ... any time you construct something on the computer then, with a lesser or greater degree of awareness, you will also construct something in your head." [33, p v]

Wenzelburger postulated a constructivist model of learning and designed study with two randomly chosen groups of third year high school students. One group studied trigonometric functions using interactive computer software, and the other in a traditional lecture environment. The main point of interest is that in addition to pre- and post-tests, there was also a delayed test given four months after the end of the five week course of study. While the computer group performed "somewhat better" than the lecture group on the immediate post-test, they performed "much better" than the lecture group on the delayed retention test. [36]

Summary

"All theory, dear friend, is gray,
but the golden tree of actual life springs ever green."

...Johann Wolfgang von Goethe

During the four years that students are undergraduate at universities in Japan, the number of hours that they spend on any one topic is extremely limited. It is important that we take the best advantage possible in preparing these students for life in the workplace after graduation. We cannot predict the nature of this workplace in detail, both because students will go on to a great variety of jobs, and the natures of those jobs are changing. However, we can be sure that more and more of those workplaces will revolve around personal desktop computers. We can be sure that the presence and power of software that puts mathematical or statistical analytical power into the hands of both experts and novices will continue to increase. It is therefore reasonable to suppose that the ability to make intelligent and creative use of these resources will also be increasingly important.

The lecture-based mathematics education that most of our students experienced before coming to university is not sufficient to prepare them to do this. It is necessary that they have active, constructive experience applying their knowledge. Without this, they will neither understand what they have studied, nor be able to use it in any useful sense.
The Laboratory may be more Effective than the Lecture

Many departments have primarily students who have opted out of mathematics early in high school. There seems to be a widespread perception among many faculty of such departments, that these students are "bad at mathematics" and any attempt to teach mathematics is doomed to failure. If we restrict ourselves to lecture based, "theorem and proof" courses, this might be correct. But, the explosion in both computers and software that does mathematics on the computer will only continue. It not only changes the nature of how we teach, it changes the nature of how we "do" mathematics, and how we use it for other fields. There is no reason to expect that the same people who are successful in the traditional style of "doing" mathematics will be the same ones who succeed using the computer.

In addition, there will soon be a new generation "Brought up from early childhood on a diet involving MTV, Nintendo, graphical calculators packed with algorithms, Macintosh-style computers, and, in the not-too-distant-future, hypermedia educational tools as well." [40]

Among these people, we may find many who, though unsuited to the lecture style, thrive in symbiosis with the computer. "...it could well be that the present dropout rate ... is a simple consequence of the mismatch between the skill currently required to succeed at school mathematics (still largely algorithmic) and those skills required to succeed at college mathematics (analytic thought, problem-solving, ...)") [40]

Our primary concern should be with what the student takes away with him or her after graduation. Indeed, it has been said that "Your education is what is left after you throw away your notes and forget everything you ever knew."[40] The laboratory, and work in groups, is a model of education that will far better insure the student still has something years after graduation, when those notes are gone.

In the the words of Alfred North Whitehead, written nearly 100 years ago,

"The solution I am urging is to eradicate the fatal disconnection of topics which kills the vitality of our modern curriculum. There is only one subject matter, and that is Life in all its manifestations. Instead of this single unity, we offer children Algebra, from which nothing follows. ."
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References


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